Notes on uniting elements method and Cayley extensions

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Article Antampoufis, Spartalis and Vougiouklis : Fundamental relations in special extensions

Method Corsini and Vougiouklis : From groupoids to groups through hypergroups. Rendiconti di Matematica VII, 9 (1989),p 173-181

Notations G_4 : quaternion group, G_8 : octonion group

$(S^n \times \mathbb{Z}_2, *)$ where S : s G_4 or G_8, $n > 1$ and finite.

$(a, b) * (c, d) = (a.c, b + d)\ \forall (a, b), (c, d) \in S^n \times \mathbb{Z}_2$

$U = \{\bar{x} \mid x \in S^n \times \mathbb{Z}_2\}$ is the set of any partition of $(S^n \times \mathbb{Z}_2, *)$ for which two elements of $(S^n \times \mathbb{Z}_2, *)$ are put together, in the same partition class.

If $\bar{x} \circ \bar{y} = \{\bar{z} \in U \mid z \in \bar{x}.\bar{y}\}\ \forall \bar{x}, \bar{y} \in U$ then $(U, \circ)$ is a H_v-group.

Theorem 1. *Uniting the elements s and $-s$ of S^n then the fundamental group $S^n/\beta^* = (\mathbb{Z}_2)^n$ if $S = G_4$ and $S^n/\beta^* = (\mathbb{Z}_2)^{2n}$ if $S = G_8$.*

Theorem 2. *Uniting the elements (s, p), $(-s, p)$ of $S^n \times \mathbb{Z}_2$ then the fundamental group U/β^* is isomorphic to $S^n/\beta^* \times \mathbb{Z}_2$.*

Theorem 3. *Uniting the elements (x, p), (y, p) of $S^n \times \mathbb{Z}_2$, $y \neq \pm x$ the fundamental group U/β^* is isomorphic to S^n/β^*.*

Theorem 4. *Uniting the elements $(s, 0)$, $(s, 1)$ of $S^n \times \mathbb{Z}_2$, the fundamental group U/β^* is isomorphic to S^n.*

Theorem 5. *Uniting the elements $(x, 0)$, $(y, 1)$ of $S^n \times \mathbb{Z}_2$, $y \neq \pm x$ the fundamental group U/β^* is isomorphic to S^n/β^*.*

This set of theorems proves de facto that the method of uniting elements via Cayley-Dickson method can be generalized to others groups.