Hyperstructures and Automorphism Groups

R. BAYON, N. LYGEROS

January 24, 2006

keywords : Automorphism groups, Hypergroups, Hyperstructures, H_v-groups, H_m-groups, H_M-groups, Moufang Identity, Rigidity

A.M.S. Classification – 20N20 – 06A11

Abstract

We first construct all the 14 rigid quasigroups of order n for $n > 2$. We exhaustively study them in order to caracterize all rigid hyperstructures. Thus we show that there are 6 rigid hypergroups for $n > 2$, 13 rigid H_v-groups of order 3 and 14 rigid H_v-groups of order $n > 3$. Finally we prove that there are 10 rigid H_m-groups of order $n > 2$. We also recall our results on enumeration of hyperstructures which are validated by our caracterization of rigid hyperstructures.

Introduction and Definitions

Definition 1. *An hypergroupoid* $< H, . >$ *is a set* H *equipped with an hyperoperation* $(.) : H \times H \longrightarrow \mathcal{P}(H)$.

Definition 2. *A quasigroup is an hypergroupoid verifying the axiom of reproduction :* $\forall x \in H \; xH = Hx = H$.

Definition 3 (F. Marty [22, 23, 24])**.** *An hypergroup* $< H, . >$ *is a quasigroup verifying associativity :* $\forall x, y, z \in H \; x(yz) = (xy)z$.

Definition 4 (Th. Vougiouklis [29])**.** $< H, . >$ *is a* H_v*-group if the following axioms hold :*

(i) $x(yz) \cap (xy)z \neq \emptyset$ *for all* x, y, z *in* H *(weak associativity)*

(ii) $xH = Hx = H$ *for all* x *in* H *(reproduction)*

Definition 5 (Th. Vougiouklis [30]). *An hyperoperation* (.) *is called smaller than the hyperoperation* ($*$)*, and written as* $. < *$*, if and only if there is an* $f \in Aut(H, *)$ *such that* $xy \subseteq f(x * y)$ *for all* x, y *in* H. He defines too the notion of minimality [32] : An hyperoperation is called minimal if it contains no other hyperoperation defined on the same set. So we can construct posets defined on set of hyperstructures.

Theorem 1 (Th. Vougiouklis [30, 31]). *A greater hyperoperation than the one of a given* H_v*-group defines a* H_v*-group.*

Definition 6 (R. Bayon - N. Lygeros [1, 18, 19]). *An hyperstructure* $< H, . >$ *is called a Marty-Moufang hypergroup and noted* H_m*-group if the reproduction axiom is valid and* (.) *verifies the Moufang identity [26, 27] :* $(xy)(zx) = x((yz)x)$.

Remark 1. $(H, bc, ac, ac, bc, ab, bc, a, a)$ *is a* H_m*-group but it is not a* H_v*-group :* $c(bb) = \{a\}$ *and* $(cb)b = \{b, c\}$.

Definition 7. *An hyperstructure* $< H, . >$ *is called a weak Marty-Moufang hypergroup and noted* H_M*-group if the reproduction axiom is valid and* (.) *verifies the weak Moufang identity :* $(xy)(zx) \cap x((yz)x) \neq \emptyset$.

Definition 8 ([11, 13, 14, 15, 16, 21]). $< H, . >$ *is a rigid hypergroupoid if and only if for all* $f \in S_n$ *and* $\forall x, y \in H$ $f(x).f(y) = f(x.y)$.

Definition 9. *Let be (.) and (*) two hyperoperations on* H *we said (*) is dual of (.) if and only if* $\forall x, y \in H x.y = y * x$.

1 Quasigroups and Rigidity

1.1 Some Preliminary Results

Fact 1. H *is a quasigroup if and only if* $d(H)$ *is a quasigroup.*

Fact 2. H *is a* H_v*-group if and only if* $d(H)$ *is a* H_v*-group.*

Fact 3. H *is a* H_m*-group if and only if* $d(H)$ *is a* H_m*-group.*

Fact 4. H *is an hypergroup if and only if* $d(H)$ *is an hypergroup.*

Fact 5. *If (.) is rigid and (*) is dual of (.) then (*) is rigid.*

Proof. By contradiction : suppose (*) non rigid i.e. $\exists f \in S_n$ such that $f(x) * f(y) \neq f(x * y)$ this implies $f(y).f(x) \neq f(y.x)$(because $\forall x, y \; x.y = x * y$) : contradiction with rigidity of (.). □

Proposition 1. *Let be $< H, . >$ a rigid quasigroup then all squares have same length and all cross products have same length.*

Proof. By contradiction : Let be $< H, . >$ a rigid quasigroup such that $\exists (x, y) \in H^2 / \, x \neq y$ and $xx = S \; yy = S'$ with $|S| \neq |S'|$. Let

$$f : \begin{cases} x \mapsto y \\ y \mapsto x \\ z \mapsto z \;\; for \; z \neq x \; and \; z \neq y \end{cases}$$

When applying f to the Cayley table of H, we obtain another Cayley table such that $xx = f(S')$ et $yy = f(S)$. However $|f(S)| = |S|$ and $|f(S')| = |S'|$, so the resulting Cayley table is different from the first one. This contradicts rigidity of H. We similarly prove this result for cross product. □

Corollary 1. *If H is a rigid quasigroup and there exists $x \in H$ such that $xx = H$ then $\forall x \in H \; xx = H$.*

Definition 10. *Let be $x.y$ an hyperproduct, completing $x.y$ such that $x.y = H$ is called completion of $x.y$.*

Definition 11. *Let be $< H, . >$ a quasigroup, we note $< H, \bar{.} >$ the quasihypergroup obtained by completion of all cross products of H.*

Definition 12. *Let be $< H, . >$ a quasigroup, we note $< H, \tilde{.} >$ the quasigroup obtained by completion of squares of H.*

Fact 6. *If $< H, . >$ is a quasigroup then $< H, \bar{.} >$ and $< H, \tilde{.} >$ are quasigroups.*

Fact 7. *If $< H, . >$ is a H_v-group then $< H, \bar{.} >$ and $< H, \tilde{.} >$ are H_v-groups.*

Proposition 2. *If $< H, . >$ is a rigid quasigroup then $< H, \bar{.} >$ is a rigid quasigroup.*

Proof. By contradiction : Suppose $< H, . >$ rigid and $< H, \bar{.} >$ non rigid.
$< H, \bar{.} >$ non rigid i.e. $\exists f : H \rightarrow H / f(x \bar{.} y) \neq f(x) \bar{.} f(y)$
but if $x \neq y$ $x \bar{.} y = H = f(H) = f(x \bar{.} y) = f(x) \bar{.} f(y)$
this implies $\exists f : H \rightarrow H / f(x \bar{.} x) \neq f(x) \bar{.} f(x)$ for some x of H.
Contradiction with rigidity of $< H, . >$. □

Proposition 3. *If $< H, . >$ is a rigid quasigroup then $< H, \tilde{.} >$ is a rigid quasigroup.*

Proof. Similar as previous proposition. □

1.2 Rigid Quasigroups

Fact 8. *The following hypergroupoids $< H, . >$ are not quasigroups :*

- $xx = x$ *and* $xy = x$
- $xx = x$ *and* $xy = y$
- $xx = H - \{x\}$ *and* $xy = x$
- $xx = H - \{x\}$ *and* $xy = y$
- $xx = H - \{x\}$ *and* $xy = H - \{x\}$
- $xx = H - \{x\}$ *and* $xy = H - \{y\}$
- $xx = H - \{x\}$ *and* $xy = H - \{x, y\}$

Proposition 4. *If $< H, . >$ is a rigid quasigroup with $|H| > 2$, there exists only three possible squares :*

- $\forall x \in H,\ xx = x$
- $\forall x \in H,\ xx = H - \{x\}$
- $\forall x \in H,\ xx = H$

Proof. If there exists x of H such that $x \in xx$ then, by transposition, for all x of H, x belongs to xx.

In the same way, if there exists x of H such that $x \notin xx$ then for all x of H, x does not belong to xx.

If there exists $y \neq x$, such that y belongs to xx. Let be z different from x and y; $xx = S \cup y$ and suppose that z does not belong to S. Let be f the transposition of y and z, then f induces a new labeling of H ($xx = S \cup z$ because $f(S) = S$) : that contradicts the rigidity of H. Consequently if there exists y different from x with $y \in xx$, then all y different from x belongs to xx. □

Proposition 5. *If $< H, . >$ is a rigid quasigroup with $|H| > 2$, there exists only seven possible cross products :*

(i) $\forall x, y \in H\ (x \neq y),\ xy = x$

(ii) $\forall x, y \in H\ (x \neq y),\ xy = y$

(iii) $\forall x, y \in H\ (x \neq y),\ xy = H - \{x\}$

(iv) $\forall x, y \in H \ (x \neq y), \ xy = H - \{y\}$

(v) $\forall x, y \in H \ (x \neq y), \ xy = H - \{x, y\}$

(vi) $\forall x, y \in H \ (x \neq y), \ xy = \{x, y\}$

(vii) $\forall x, y \in H \ (x \neq y), \ xy = H$

Proof. Suppose there exists $(x, y) \in H^2$ with $x \neq y$ and $x \in xy$. Let z in H and f be the transposition of x and z, then z is in zy because of definition 6. This time, let be f the transposition of z and y, then x is in xz. So if there exists a (x, y) in H^2 (with $x \neq y$) such that $x \in xy$ then for all (x, y) in H^2 (with $x \neq y$), x belongs to xy. In the same way we show the following results :

- if there exists a (x, y) in H^2 (with $x \neq y$) such that $y \in xy$ then for all (x, y) in H^2 (with $x \neq y$), y belongs to xy.
- if there exists a (x, y) in H^2 (with $x \neq y$) such that $x \notin xy$ then for all (x, y) in H^2 (with $x \neq y$), x does not belong to xy.
- if there exists a (x, y) in H^2 (with $x \neq y$) such that $y \notin xy$ then for all (x, y) in H^2 (with $x \neq y$), y does not belong to xy.

Let $\alpha \in H$ with $\alpha \neq x$ and $\alpha \neq y$, so by rigidity α is in xy (using the transposition of α and z). So $H - \{x, y\} \in xy$ and, by combination of proposition 1 and previous result, this implies that if there exists a (x, y, z) in H^3 with $x \neq y$, $x \neq z$ and $y \neq z$ such that $z \in xy$ then $\forall x, y \ H - \{x, y\} \subset xy$.

The combination of the five previous results proves the current proposition. □

We summarize our results in table 1.

	x.y						
x.x	x	y	$H - \{x\}$	$H - \{y\}$	$H - \{x, y\}$	$\{x, y\}$	H
x	-	-	H_{v1}	$d(H_{v1})$	Q_1	H_1	H_2
$H - \{x\}$	-	-	-	-	-	H_3	H_4
H	H_{v2}	$d(H_{v2})$	H_{v3}	$d(H_{v3})$	H_{v4}	H_5	H_6

Table 1: Rigid Quasigroups

2 An Exhaustive Study of Rigid Quasigroups

We now precise the nature of rigids quasigroups.

Proposition 6. *H_{v1} and $d(H_{v1})$ are H_v-groups.*

Proof. H_{v1} is not an hypergroup : $x(yy) = xy = H - \{x\}$ and $(xy)y = H - \{x\}.y \supset zy = H - \{z\} \ni x$.
If x, y, z are all different, $x(yz) = x.H - \{y\} \supset x.\{x, z\} = \{x\} \cup H - \{x\} = H$.
If $x \neq y$, $(xx)y = xy = H - \{x\}$ and $x(xy) = x.H - \{x\} \supset xy = H - \{x\}$.
If $x \neq y$, $(xy)x = H - \{x\}.x \supset y.x \ni x$ and $x(yx) = x.H - \{y\} \supset xx = x$.
If $x \neq y$, $x(yy) = xy = H - \{x\} \ni y$ and $(xy)y = H - \{x\}.y \ni y$.

Thanks to facts 2 and 4, $d(H_{v1})$ is an H_v-group and is not an hypergroup. □

Proposition 7. *H_{v2} and $d(H_{v2})$ are H_v-groups.*

Proof. H_{v2} is not an hypergroup : if $x \neq y$, $(yx)y = yy = H \, and \, y(xy) = yx = y$.
$\forall (x, y, z) \in H^3$, $(xy)z \supset xz \supset \{x\}$ and $x(yz) \supset xy \supset \{x\}$.
So H_{v2} is an H_v-group.

Thanks to facts 2 and 4, $d(H_{v2})$ is an H_v-group and is not an hypergroup. □

Proposition 8. *H_{v3} and $d(H_{v3})$ are H_v-groups.*

Proof. H_{v3} is not an hypergroup : if $x \neq y$, $(xx)y = Hy = H \, and \, x(xy) = x.H - \{x\} \not\ni x$.
So H_{v3} is an H_v-group (by completion of squares of H_{v1}).

Thanks to facts 2 and 4, $d(H_{v3})$ is a H_v-group and is not an hypergroup. □

Proposition 9. *H_{v4} and $d(H_{v4})$ are H_v-groups.*

Proof. If x, y, z are all different : $(xy)z = H - \{x, y\}.z = H$, then $(xy)z \cap x(yz) \neq \emptyset$.
If $x \neq y$, $(xx)y = H$ then $(xx)y \cap x(xy) \neq \emptyset$.
If $x \neq y$, $x(yy) = H$ then $(xy)y \cap x(yy) \neq \emptyset$.
If $x \neq y$, then $(xy)x = (yx)x = x(yx)$ (because H_{v4} is abelian).
H_{v4} is not an hypergroup : if $x \neq y$, $x(yy) = H$ and $(xy)y = H - \{x, y\}.y \not\ni y$.

Thanks to facts 2 and 4, $d(H_{v4})$ is not an hypergroup and is a H_v-group. □

Proposition 10. *Q_1 is a quasigroup at order 3 and a H_v-group at order greater than 3.*

Proof. For $H = \{x, y, z\}$, Q_1 is not a H_v-group : if x, y, z are all different $(xy)z = zz = z$ and $x(yz) = xx = x$.
For $H \supset \{x, y, z, \alpha\}$, Q_1 is a H_v-group.
If x, y, z are all different : $x(yz) = x.H - \{y, z\} \supset x.\{x, \alpha\} = x \cup x.\alpha \supset H - \{x, \alpha\} \supset y$ and $(xy)z = H - \{x, y\}.z \supset \{z, \alpha\}.z = z \cup \alpha z \supset H - \{\alpha, z\} \supset y$.
If $x \neq y$, $(xx)y = xy = H - \{x, y\}$ and $x(xy) = x.H - \{x, y\} \supset x.\{z, \alpha\} = H - \{x, z\} \cup H - \{x, \alpha\} \supset \{z, \alpha\}$.
If $x \neq y$, $(xy)y = x(yy)$ with previous result and because Q_1 is abelian.
If $x \neq y$, $(xy)x = x(xy) = x(yx)$.

□

3 Hypergroups

Proposition 11. *If $< H, . >$ is an abelian rigid hypergroup with $|H| > 2$ then there exists two types of possible cross products:*

- $xy = \{x, y\}$
- $xy = H$

Proof. If there exists $z \in H$ so that $z \neq x$, $z \neq y$ and $z \in xy$, so by rigidity for any xy cross product $H - \{x, y\} \subset xy$ (For that, consider the α different from x,y,z and the (z, α) transpositions).

Let us suppose there exist x, y with $x \neq y$ so that $x \notin xy$ so, with the previous assumption, we have for any $x \neq y$ $H - \{x, y\} = xy$ (indeed, if $x \notin xy$ by commutativity $y \notin xy$ et $xy \neq \emptyset$ hyperoperation's definition). Now let us consider every possible squares :

- $xx = H$

  Let's compute $(xx)y = Hy$ or $xy = H - \{x, y\}$, so $Hy \not\ni y$ which contradicts the reproduction.

- $xx = x$

  $(xx)y = xy = H - \{x, y\}$ and $x(xy) = x.H - \{x, y\}$ so $H - \{x, y\} \notin x(xy)$ which contradicts the associativity.

- $xx = H - \{x\}$

  $(xx)y = H - \{x\}.y$ so $(xx)y = \{x\}$ or $(xx)y = \emptyset$ $x(xy) = x.H - \{x, y\}$ and so $x \notin x(xy)$ which contradicts the associativity.

So we have shown that for every cross products $\{x, y\} \subset xy$. By exploiting our first affirmation we have then two types of cross products: either, for every xy, $xy = H$, or, for every xy, $xy = \{x, y\}$. □

We know that non-abelian rigid quasigroups are not hypergroups. So we can verify, via the combination of all the square products with all the cross products, that H_1, H_2, H_3, H_4, H_5, H_6 are hypergroups.

Proposition 12. *H_1, H_2, H_3, H_4, H_5 and H_6 are hypergroups.*

4 H_m-groups

Proposition 13. *Q_1, H_{v2} and H_{v4} are not H_m-groups.*

Proof. Q_1 : If $Q_1 = \{x, y, z\}$ and $x \neq y$, $(xy)(yx) = zz = z$ and $x((yy)x) = x(yx) = xz = y$
If $Q_1 \supset \{x, y, z, \alpha\}$ $(xy)(yx) \supset \{z, \alpha\}.\{z, \alpha\} \supset z\alpha = H - \{z, \alpha\} \ni x$ and $x((yy)x) = x.H - \{x, y\} \not\ni x$.
H_{v2} : if $x \neq y$, $(xy)(xx) = H$ and $x((yx)x) = x(yx) = xy = x$
H_{v4} : if $x \neq y$, $(xx)(yx) = H$ and $x((xy)x) = x(H - \{x, y\}.x)$, or $x \notin (H - \{x, y\}.x$ so $x \notin x((xy)x)$. □

Proposition 14. *H_{v1} and $d(H_{v1})$ are H_m-groups.*

Proof. If x, y, z are all different, $(xy)(zx) = H - \{x\}.H - \{z\} \supset y.\{x, y\} = H$ and $x((yz)x) = x(H - \{y\}.x) \supset x.(\{x, z\}.x) \supset xH = H$.
If $x \neq y$, $(xx)(yx) = x.H - \{y\} \supset x.\{x, z\} = H$ and $x((xy)x) = x(H - \{x\}.x) \supset x(\{y, z\}.x) = x(H - \{y\} \cup H - \{z\}) = H$
If $x \neq y$, $(xy)(xx) = H - \{x\}.x \supset \{y, z\}.x = H - \{y\} \cup H - \{z\} = H$ and $x((yx)x) = x(H - \{y\}.x) \supset x(\{x, z\}.x) = x(xx \cup zx) = x.H - \{z\} \supset x.y \cup xx = H$
If $x \neq y$, $(xy)(yx) = H - \{x\}.H - \{y\} \supset z.\{x, z\} = H$ and $x((yy)x) = x(yx) = x.H - \{y\} \supset x.\{x, z\} = H$
Thanks to facts 3, $d(H_{v1})$ is a H_m-group. □

Proposition 15. *H_{v3} and $d(H_{v3})$ are H_m-groups.*

Proof. If x, y, z are all different, $(xy)(zx) = H - \{x\}.H - \{z\} \supset yy = H$ and $x((yz)x) = x(H - \{y\}x) = xH = H$
If $x \neq y$, $(xx)(yx) = H$ and $x((xy)x) = x(H - \{x\}.x) \supset x(yx) = x.H - \{y\} \supset xx = H$

If $x \neq y$, $(xy)(xx) = H$ and $x((yx)x) = x(H - \{y\}.x) \supset x(xx) = H$
If $x \neq y$, $(xy)(yx) = H - \{x\}.H - \{y\} \supset zz = H$ and $x((yy)x) = H$.

Thanks to facts 3, $d(H_{v3})$ is a H_m-group. □

5 Enumeration

In the enumeration theory we already obtained some results in different fields [7, 8, 9, 10, 20]. And in our previous work we enumerate and classify the hypergroups of order 3 [3, 5] and abelian hypergroups of order 4 [4]. We then study the H_v-groups of order 3 [6], and abelian H_v-groups of order 4 with Marty-Moufang hypergroups [1].

Thanks to these enumerative results we can caracterize some hyperstructures, has shown here with rigid hyperstructures or with hypocomplete hypergroups [3, 17]. The obtained results contribute consequently to validate our algorithm. We could confirm the results of R. Migliorati [25], some results of S-C. Chung and B-M Choi [12] and some results of Th. Vougiouklis [32, 33] too. We present now, the best computational results in this fields.

Theorem 2 (G. Nordo [28])**.** *There are 3.999 isomorphism classes of hypergroups of order 3 (see table 2).*

Theorem 3 (R. Bayon - N. Lygeros [4])**.** *There are 10.614.362 isomorphism classes of abelian hypergroups of order 4 (see table 3).*

Theorem 4 (R. Bayon - N. Lygeros [6])**.** *There are 20 isomorphism classes of H_v-groups of order 2 (see table 4).*

Theorem 5 (R. Bayon - N. Lygeros [6])**.** *There are 1.026.462 isomorphism classes of H_v-groups of order 3 (see table 5).*

Theorem 6 (R. Bayon - N. Lygeros [2])**.** *There are 8.028.299.905 isomorphism classes of abelian H_v-groups of order 4 (see table 6).*

Theorem 7 (R. Bayon - N. Lygeros [1])**.** *There are 10 isomorphism classes of Marty-Moufang hypergroups of order 2 (see table 7).*

Theorem 8 (R. Bayon - N. Lygeros [1])**.** *There are 96.058 isomorphism classes of H_m-groups of order 3 (see table 8).*

		Classes					
		Abelians			non Abelians		
		Cyclics	non Cyclics Proj.	non Cyclics non Proj.	Cyclics	non Cyclics Proj.	non Cyclics non Proj.
$\lvert Aut(H)\rvert$	1	4	2	-	-	-	-
	2	3	-	-	6	1	-
	3	70	3	5	154	8	4
	6	360	2	17	3279	20	61

Table 2: Classification of hypergroups of order 3

		Classes		
		Cyclics	non Cyclics Proj.	non Cyclics non Proj.
$\lvert Aut(H)\rvert$	1	4	2	-
	2	-	-	-
	3	14	2	2
	4	162	7	13
	6	312	5	20
	8	246	-	4
	12	37.426	54	801
	24	10.569.502	53	5.733

Table 3: Classification of abelian hypergroups of order 4

H_v-group	$\lvert Aut(H_v)\rvert$	H_v-group	$\lvert Aut(H_v)\rvert$
$(a;b;b;a)^*$	2	$(H;a;H;b)^*$	2
$(H;b;b;a)$	2	$(a;H;H;b)^*$	1
$(a;H;b;a)$	2	$(H;a;a;H)$	2
$(a;b;H;a)$	2	$(H;b;a;H)$	1
$(H;a;a;b)^*$	2	$(H;a;b;H)$	1
$(H;H;b;a)$	2	$(H;H;H;a)^*$	2
$(H;b;H;a)$	2	$(H;H;H;b)^*$	2
$(a;H;H;a)$	2	$(H;H;a;H)$	2
$(b;H;H;a)$	1	$(H;H;b;H)$	2
$(H;H;a;b)^*$	2	$(H;H;H;H)^*$	1

Table 4: H_v-groups of Order 2 ($H = \{a,b\}$)

		Classes					
		Abelian			non Abelians		
		Cyclics	non Cyclics		Cyclics	non Cyclics	
			Proj.	non Proj.		Proj.	non Proj.
$\|Aut(H_v)\|$	1	5	2	-	4	2	-
	2	8	1	1	47	5	7
	3	243	8	14	2034	66	76
	6	7439	10	195	1003818	1083	11394

Table 5: Classification of H_v-groups of order 3

		Classes		
		Cyclics	non Cyclics	
			Proj.	non Proj.
$\|Aut(H_v)\|$	1	5	3	-
	2	-	-	-
	3	38	5	6
	4	582	22	39
	6	2.215	45	144
	8	2.149	39	144
	12	1.859.161	1.827	39.773
	24	7.994.020.227	86.159	32.287.322

Table 6: Classification of abelian H_v-groups of Order 4

H_m-group	$\|Aut(H_m)\|$
$(a; b; b; a)^*$	2
$(H; H; H; a)^*$	2
$(H; a; a; b)^*$	2
$(H; H; a; b)^*$	2
$(H; a; H; b)^*$	2
$(a; H; H; b)^*$	1
$(H; H; H; b)^*$	2
$(H; H; a; H)$	2
$(H; H; b; H)$	1
$(H; H; H; H)^*$	1

Table 7: Isomorphism classes of Marty-Moufang Hypergroups of order 2.

$\lvert Aut(H_m)\rvert$	1	2	3	6
	10	30	770	95.248

Table 8: Number of Marty-Moufang Hypergroups isomorphism classes relatively to the order of their automorphism groups

Acknowledgements

We thank Ph. Alsina, P. Deloche and Y. Martinez for their precious help.

References

[1] R. Bayon and N. Lygeros. The H_v-groups and Marty-Moufang Hypergroups. In *Proceedings of First International Conference on Algebraic Informatics*, pages 285–294. Aristotle University of Thessaloniki, 2005.

[2] R. Bayon and N. Lygeros. Number of abelian H_v-groups of order n. In N. J. A. Sloane, editor, *The On-Line Encyclopedia of Integer Sequences*, http://www.research.att.com/projects/OEIS?Anum=A108089, 2005.

[3] R. Bayon and N. Lygeros. Catégories spécifiques d'hypergroupes d'ordre 3. In *Eléments structurels de la théorie des hyperstructures: Colloque de l'Université de Thrace*, march 2005.

[4] R. Bayon and N. Lygeros. Les hypergroupes abéliens d'ordre 4. In *Eléments structurels de la théorie des hyperstructures: Colloque de l'Université de Thrace*, march 2005.

[5] R. Bayon and N. Lygeros. Les hypergroupes d'ordre 3. *Italian Journal of Pure and Applied Mathematics*, to appear.

[6] R. Bayon and N. Lygeros. Les hypergroupes et H_v-groupes d'ordre 3. *Bulletin of the Greek Mathematical Society*, to appear.

[7] R. Bayon, N. Lygeros, and J.-S. Sereni. Nouveaux progrès dans l'énumération des modèles mixtes. In *Knowledge discovery and discrete mathematics: JIM'2003*, pages 243–246, Université de Metz, France, 2003. INRIA.

[8] R. Bayon, N. Lygeros, and J.-S. Sereni. New progress in enumeration of mixed models. *Appl. Math. E-Notes*, 5:60–65, 2005.

[9] C. Chaunier and N. Lygeros. The number of orders with thirteen elements. *Order*, 9(3):203–204, 1992.

[10] C. Chaunier and N. Lygeros. Le nombre de posets à isomorphie près ayant 12 éléments. *Theoret. Comput. Sci.*, 123(1):89–94, 1994. Number theory, combinatorics and applications to computer science (Marseille, 1991).

[11] C. Chaunier and N. Lygeros. Posets minimaux ayant un groupe d'automorphismes d'ordre premier. *C. R. Acad. Sci. Paris Sér. I Math.*, 318(8):695–698, 1994.

[12] S-C. Chung and B-M Choi. H_v-groups on the set $\{e, a, b\}$. *Italian Journal of Pure and Applied Mathematics*, 10:133–140, 2001.

[13] N. Lygeros. Caractérisation de la complexit structurelle. *Perfection*, 1, 2002.

[14] N. Lygeros. Posets, groups and hypergroups. *Perfection*, 5, 2004.

[15] N. Lygeros. Sur la notion de groupe d'automorphismes. *Perfection*, 8, 2004.

[16] N. Lygeros. Sur les hypergroupes rigides. *Perfection*, 8, 2004.

[17] N. Lygeros. Hypergroupes et groupes d'automorphismes. *Perfection*, 1, 2005.

[18] N. Lygeros. Les hypergroupes de Marty-Moufang. *Perfection*, 6, 7 2005.

[19] N. Lygeros. Sur les hypergroupes de Marty-Moufang d'ordre 2. *Perfection*, 6, 7 2005.

[20] N. Lygeros, P.V. Marchand, and M. Massot. Enumeration and 3d-representation of the stereo-isomers of paraffinic molecules. *Journal of Symbolic Computation*, 40:1225–1241, 2005.

[21] N. Lygeros and M. Mizony. Construction de posets dont le groupe d'automorphismes est isomorphe à un groupe donné. *C. R. Acad. Sci. Paris Sér. I Math.*, 322(3):203–206, 1996.

[22] F. Marty. Sur une généralisation de la notion de groupe. In *8ème congrès des Mathématiciens Scandinaves, Stockholm*, pages 45–49, 1934.

[23] F. Marty. Rôle de la notion d'hypergroupe dans l'étude des groupes non abéliens. *C. R. Acad. Sci. Paris Math.*, 1935.

[24] F. Marty. Sur les groupes et hypergroupes attachés à une fraction rationnelle. *Annales scientifiques de l'E.N.S.*, 53:83–123, 1936.

[25] R. Migliorato. Ipergruppi di cardinalità 3 e isomorfismi di ipergruppoidi commutativi totalmente regolari. *Atti Convegno su Ipergruppi, altre Strutture Multivoche e loro applicazioni, Udine*, 1985.

[26] R. Moufang. Die Desarguesschen Sätze vom Rang 10. *Math. Ann.*, 108:296–310, 1933.

[27] R. Moufang. Zur Struktur von Alternativkörpern. *Math. Ann.*, 110:416–430, 1935.

[28] G. Nordo. An algorithm on number of isomorphism classes of hypergroups of order 3. *Italian Journal of Pure and Applied Mathematics*, 1995.

[29] Th. Vougiouklis. The fundamental relation in hyperrings: The general hyperfield. pages 203–211, 1991.

[30] Th. Vougiouklis. *Hyperstructures and their Representations*. Hadronic Press, 1994.

[31] Th. Vougiouklis. A new class of hyperstructures. *Journal of combinatorics, information & system sciences*, 20:229–235, 1995.

[32] Th. Vougiouklis. H_v-groups defined on the same set. *Discrete Mathematics*, 155:259–265, 1996.

[33] Th. Vougiouklis, S. Spartalis, and M. Kessoglides. Weak hyperstructures on small sets. *Ratio Mathematica*, 12:90–96, 1997.