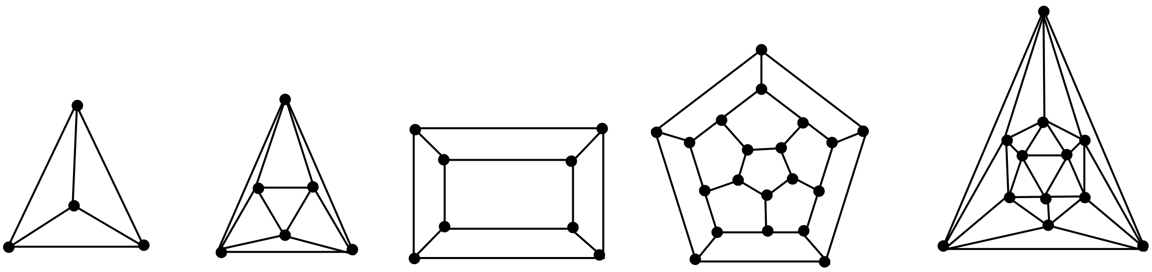
Line graphs and Platonic graphs

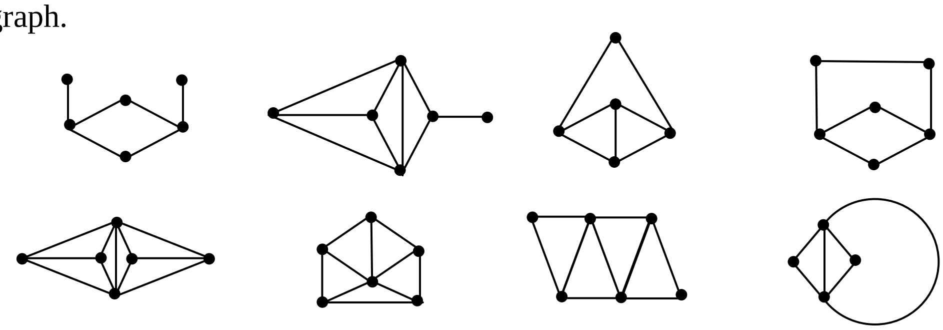
N. Lygeros, I. Pitault

A Platonic graph is a graph which corresponds to the skeleton of a Platonic solid. So we have five Platonic graphs which are special cases of polyhedral graphs. An n-polyhedral graph is a 3-connected simple planar graph on n nodes. It is known that every convex polyhedron can be represented in the plane by a 3-connected planar graph. We have the same property on the surface of a sphere. And we have also the converse via the Theorem of Steinitz.



Tetrahedral graph   Octahedral graph   Cubical graph   Dodecahedral graph   Isocahedral graph

A line graph of an undirected graph G is another graph L(G) that represents the adjacencies between edges of G. A nice characterization of line graphs was proven by Beineke. A graph is a line graph if and only if it does not contain one of them following nine graphs as an induced subgraph.



Now if we use line graphs to study Platonic graphs we obtain a different result from the duality approach, because we get on one hand:

$T \circlearrowleft$, $C \longleftrightarrow O$, $D \longleftrightarrow I$ (Duality)

and on the other:

$T \longrightarrow O$, $D \longrightarrow I$, $C \longrightarrow CO$

where CO is the cuboctahedral graph