Graphs generated by the iteration of line graph operations

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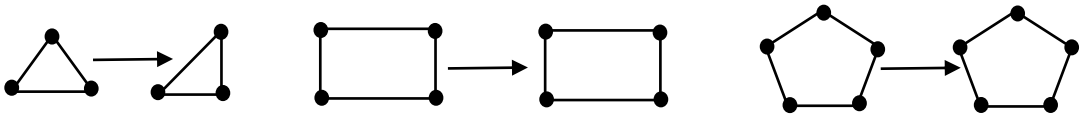
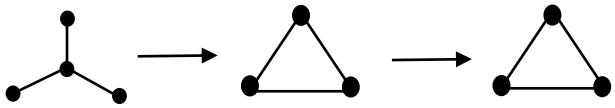
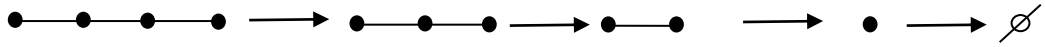
A line graph of an undirected graph G is another graph L(G) that represents the adjacencies between edges of G. So we can define an operation: $G \xrightarrow{L} L(G)$, and its iteration: $L \circ L, L \circ L \circ L, \ldots$ We get the following sequence of graphs:
G, L(G), L(L(G)), L(L(L(G))), L(L(L(L(G)))), ...
To study the graphs generated by the iteration of line graph operation, we use the Theorem of van Rooij and Wilf. They have proved that when G is a finite connected graph, only four possible behaviors are possible.
1) If G is a cycle graph then L(G) is isomorphic to itself. By the way, Harary has proved that they are the only connected graphs for which L(G) is isomorphic to G.



2) If G is $K_{1,3}$ then L(G) is a triangle



3) If G is P_n then L(G) is P_{n-1}



4) Else the size of L(G) increases so the sequence can produce graphs with sizes that are not bounded.