Line graph and characteristic polynomial

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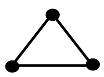
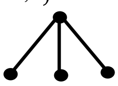
Let n the number of vertices, m the number of edges, A the adjacency matrix, D the diagonal matrix of vertex degrees, R the vertex-edge incidence matrix of a graph G and A_L the adjacency matrix of the line graph $L(G)$ of G. We have the following relations:

$$RR^T = A + D$$
$$RR^T = A_L + 2I$$

Since non-zero eigenvalues of RR^T and $R^T R$ are the same, we can deduce from the previous equalities that $P_{L(G)}(\lambda) = (\lambda + 2)^{m-n} Q_G(\lambda + 2)$ when Q_G is the characteristic polynomial of the matrix $Q = A + D$.

This relation between A and Q matrices via the notion of line graph, produces a fundamental relation between their characteristic polynomial. Even if the Laplacian matrix, i.e. L=D-A is more considered due to the history of mathematics, the singless Laplacian matrix seems to be more efficient to discriminate graphs as the known ratio of cospectral graphs of Q is less than the ratio of cospectral graphs of L. This consideration is a strong basis for the promotion of the study of Q matrix. By the way, we have a direct link between the spectra of line graphs and the $Q-$spectra of graphs.

It is also well known that if $\{G, H\} \neq \{K_3, K_{1,3}\}$

K_3  $K_{1,3}$ 

then we have the following result: if G and H are connected graphs such that $L(G) = L(H)$ then $G = H$. So we have a strategy rather different from the classical since we can study the spectrum of the line graph instead of the spectrum of the graph.

And as we know that if two graphs are $Q-$cospectral then they are $L-$cospectral but the converse is false, we have one more argument for the $Q-$matrix.