Definition of Synergetic Hyperstructures

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We can interpret the notion of group with the following necessary but not sufficient condition: $\forall(a,b)\in G^2 : |ab| = 1$. We know that in a group, we have an inner operation which means that: $\forall(a,b)\in G^2 : ab \in G$. In terms of cooperation we can say that the elements a and b cooperate to give a product. We can generalize this mental shema to define synergetic hyperstructures with the notion of hyperoperation. In a hypergroup H, with the definition of Marty, we have: $\forall(a,b)\in H^2 : 1 \leq |a*b| \leq |H|$ which is a consequence of the axioms of reproduction and associativity. With this context, we can give a more formal difference between cooperation and synergy. The group operation is a cooperation but not synergy because we have only $|ab| = 1$. To define a synergy we need more because a synergy is more than a cooperation. So we need $|a*b| > 1$. With this condition, we can define synergetic hyperstructures in which we have:

$$\forall(a,b)\in SH : |a*b| > 1$$

and strong synergetic hyperstructures:

$$\forall(a,b)\in SSH : |a*b| > 2$$

With those definitions we can see easily that a strong synergy has the following propriety:

$$a \cup b = \{a,b\} \subset a*b$$