Facts about synergetic hyperstructures

N. Lygeros

Let H an Hypergroup.
We have $\forall h \in H : h*H = H*h = H$
and $\forall (a,b,c) \in H^3 : (a*b)*c = a*(b*c)$

H is a synergetic hypergroup if and only if:

$$\forall (a,b) \in H^2 : |a*b| > 1$$

H is a strong synergetic hypergroup if and only if:

$$\forall (a,b) \in H^2 : |a*b| > 2$$

Fact 1: There is only one synergetic hypergroup of order 2.
Proof: If H is a synergetic hypergroup or order 2, we have:

$$\forall (a,b) \in H^2 : 1 < |a*b| \le 2$$

which means: $\forall (a,b) \in H^2 : |a*b| = 2$ so $\forall (a,b) \in H^2 : a*b = H$.
This implies that: $SH = (H, H, H, \mathrm{H})$

Fact 2: There is no strong synergetic hypergroup of order 2.

Proof: Consequence of fact 1.

Fact 3: There is only one strong synergetic hypergroup of order 3.

Proof: If H is a strong synergetic hypergroup of order 3, we have:
$\forall (a,b) \in H^2 : 2 < |a*b| \le 3$

which means: $\forall (a,b) \in H^2 : |a*b| = 3$ so $\forall (a,b) \in H^2 : a*b = H$.
This implies that: $SSH = (H, H, H, H, H, H, H, H, H)$
Fact 4: We have the same results for Hv-groups.
Fact 5: Synergetic hyperstructures have no scalar unit.