Carathéodory : Perron : Klammersymbole

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Perron: Klammersymbole

$f(x^i)\ \ (i = 1, \dots, n);\quad \dfrac{\partial f}{\partial x^i} = f_i$

$P^v(x^i),\ \ Q^v(x^i);\ \dfrac{\partial P^v}{\partial x^i} = P_i^v;\quad \dfrac{\partial Q^v}{\partial x^i} = Q_i^v$

$$\begin{cases} \dfrac{dz^v}{dt} = P^v(z^i) \\ z^v = \varphi^v(t, x^j) \\ x^v = \varphi^v(0, x^j) \end{cases};\qquad \begin{cases} \dfrac{dz^v}{du} = Q^v(z^i) \\ z^v = \psi^v(u, x^j) \\ x^v = \psi^v(0, x^j) \end{cases}$$

Annahme $\quad f_i P^i = 0 \quad f_i Q^i = 0$

$$f\left(\varphi^i(t, x^j)\right) = F(t, x^k)$$

$$F(0, x^k) = f(x^i)$$

$$\frac{\partial F}{\partial t} = f_i \frac{\partial \varphi^i}{\partial t} = f_i P^i = 0$$

$$f\left(\varphi^i\left(t, x^j\right)\right) = f(x^k)$$

$$f\left(\psi^i\left(u, x^j\right)\right) = f(x^k)$$

$$f\left(\varphi^i\left(u, \psi^j\right)\right) = f(x^k)$$

$$\frac{\partial f\left(\varphi^i\left(t, \psi^j\right)\right)}{\partial u} = f_m\left(\varphi^i\left(t, \psi^j\right)\right).\varphi_k^m\left(t, \psi^j\right).Q\left(u, \psi^j\right) = 0\ \mu$$

Für u=0 $\quad f_m\left(\varphi^i\left(t, x^j\right)\right).\varphi_k^m(t, x^j).Q^k\left(x^j\right) = 0$

Wir setzen :

$$\varphi^i(t, x^j) = y^i \rightarrow x^j = y^j(-t, y^i)$$

$$\Phi_k^m(t, y^i) = \varphi_k^m(t, \varphi^j(-t, y^l))$$

$$\Omega^k(t, y^i) = Q^k(\varphi^j(-t, y^l))$$

$$\frac{\partial \Omega^k}{\partial t} = -Q_j^k(\varphi^k(-t, y^l))P_f^j(y^i)$$

$$\left.\frac{\partial \Omega^k}{\partial t}\right|_{t=0} = -Q_j^k(y^i)P^j(y^i)$$

Die Funktion $\Phi_k^m(t, y^i)$ ist am Punkte t=0 nach t differentiierbar :

$$\Phi_k^m(t, y^j) - \Phi_k^m(0, y^j) = \varphi_k^m(t, \varphi^j(-t, y^j)) - \varphi_k^m(0, y^j)$$

Nun ist $\varphi_k^m(0, y^j) = \delta_k^m = \varphi_k^m(0, x^i)$

$$\begin{aligned}
\Phi_k^m(t, y^j) - \Phi_k^m(0, y^j) &= \varphi_k^m(t, x^i) - \varphi_k^m(0, x^i) \\
&= t\frac{\partial}{\partial t}\left(\varphi_k^m(\theta t, x^i)\right) \\
&= tP_j^m\left(\varphi^i(\theta t, x^l)\right)\varphi_k^j\left(\theta t, x^i\right)
\end{aligned}$$

$$\left.\frac{\partial}{\partial t}\Phi_k^m(t, y^j)\right|_{t=0} = P_j^m(y^i).\varphi_k^j(0, x^i) = P_j^m.\delta_k^j = P_k^m(y^i)$$

Nun ist $\Phi_k^m(0, y^i) = \varphi_k^m(0, y^j) = \delta_k^m$

$$\Omega^k(0, y^i) = Q^k(y^j)$$

$$\begin{aligned}
\frac{\partial}{\partial t}\left(\Phi_k^m\Omega^k\right) &= -\delta_k^m Q_j^k(y^i)P^j(y^i) + P_k^m(y^i)Q^k(y^i) \\
&= \left(P_k^m Q^k - Q_j^m P^j\right)
\end{aligned}$$

In (1) eingesetzt, erhält man $\left(P_k^m Q^k - Q_j^m P^j\right) f_m = 0$