Μέθοδος ελαχίστων τετραγώνων (παραβολική περίπτωση)

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$$y = \alpha_0 + \alpha_1 x + \alpha_2 x_2$$

$$R2 \equiv \sum_{i=1}^{n} \left[y_i - \left(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 \right) \right]^2$$

Παραγωγίζουμε προς όλες τις παραμέτρους:

$$\begin{cases} \dfrac{\partial R^2}{\partial \alpha_0} = -2\sum_{i=1}^{n} \left[y_i - \left(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 \right) \right] = 0 \\ \dfrac{\partial R^2}{\partial \alpha_1} = -2\sum_{i=1}^{n} \left[y_i - \left(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 \right) \right] x_i = 0 \\ \dfrac{\partial R^2}{\partial \alpha_2} = -2\sum_{i=1}^{n} \left[y_i - \left(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2 \right) \right] x_i^2 = 0 \end{cases}$$

$$\begin{cases} a_0 n + a_1 \sum_{i=1}^{n} x_i + a_2 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i \\ a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 + a_2 \sum_{i=1}^{n} x_i^3 = \sum_{i=1}^{n} x_i y_i \\ a_0 \sum_{i=1}^{n} x_i^2 + a_1 \sum_{i=1}^{n} x_i^3 + a_2 \sum_{i=1}^{n} x_i^4 = \sum_{i=1}^{n} x_i^2 y_i \end{cases}$$

Με τη μορφή πινάκων έχουμε:

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i^2 y_i \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$y = Xa$$

$$X^T y = X^T X a$$

$$a = \left(X^T X\right)^{-1} X^T y$$

with ($linalg$):

$X := matrix\left(\left[\left[1, x_1, x_1^2\right], ..., \left[1, x_n, x_n^2\right]\right]\right);$

$y := matrix\left(n, 1, \left[y, \cdots, y_n\right]\right);$

$\# a := matrix\left(3, 1, \left[a_0, a_1, a_2\right]\right);$

$t := transpose(X);$

$X_1 := evalm(t \,\&* X);$

$a := map(x \rightarrow simplify(x), evalm(inverse(X_1) \,\&* t \,\&* Y);$

$map(x \rightarrow evalf\,(x), a);$