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$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau_1 \tau_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i \tau_3$$

$$\tau_1 \tau_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -i \tau_2$$

$$\tau_2 \tau_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = i \tau_1$$

$$\tau_1 \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\tau_2 \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\tau_3 \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$\tau_2 \tau_1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = -i \tau_3$$

$$\tau_3 \tau_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -i \tau_2$$

$$\tau_3 \tau_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = -i \tau_1$$

$$[\tau_i, \tau_j] = \tau_i \tau_j - \tau_j \tau_i = 2\tau_i \tau_j = (-1)^{i+j+1} \tau_k \quad \{\tau_i, \tau_j\} = 0$$

$$\tau_i^2 = I$$

$$\tau_1 = [(\tau_1)_{ij}] = \begin{bmatrix} \tau_{111} & \tau_{112} \\ \tau_{121} & \tau_{122} \end{bmatrix} \tau_2 = [(\tau_2)_{kl}] = \begin{bmatrix} \tau_{211} & \tau_{212} \\ \tau_{221} & \tau_{222} \end{bmatrix}$$

$$\sum_i \sum_j \tau_i \tau_j = \tau_1 \tau_1 + \tau_1 \tau_2 + \tau_1 \tau_3$$

$$\tau_2 \tau_1 + \tau_2 \tau_2 + \tau_2 \tau_3 = \sum_i (\tau_i)^2 + \sum_{\substack{i,j \\ i \neq j}} \tau_i \tau_j = 3\mathbf{I} = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix} = 3[(\delta_{ij})]$$

$$\tau_3 \tau_1 + \tau_3 \tau_2 + \tau_3 \tau_3$$

$$A.B = (a_{ij})(b_{jk}) = \left[ \sum_{i,j,k} a_{ij} \cdot b_{jk} \right]$$

$$\begin{bmatrix} \tau_1[1,1] & \tau_1[1,2] \\ \tau_1[2,1] & \tau_1[2,2] \end{bmatrix} \cdot \begin{bmatrix} \tau_2[1,1] & \tau_2[1,2] \\ \tau_2[2,1] & \tau_2[2,2] \end{bmatrix}$$

$$\tau_1[1,1] \cdot \tau_2[1,1] + \tau_1[1,2] \cdot \tau_2[2,1] \quad \tau_1[1,1] \cdot \tau_2[1,2] + \tau_1[1,2] \cdot \tau_2[2,2]$$

$$\tau_1[2,1] \cdot \tau_2[1,1] + \tau_1[2,2] \cdot \tau_2[2,1] \quad \tau_1[2,1] \cdot \tau_2[1,2] + \tau_1[2,2] \cdot \tau_2[2,2]$$

$$\sum_i (\tau^i)_{ij} (\tau^a)_{kl} = 2 \left( \delta_{jk} \delta_{li} - \frac{i}{2} \delta_{ij} \delta_{kl} \right)$$

$$\tau_j \tau_k = \delta_{jk} + i \sum_1 \varepsilon_{jkl} \tau_1$$

$$\varepsilon_{jkl} : \begin{cases} +1, jkl=123, 231, 312 \\ -1, jkl=132, 213, 321 \\ 0 \end{cases}$$

$$\tau_1 \tau_2 = i \varepsilon_{123} \tau_3 = i \tau_3$$

$$\tau_1 \tau_3 = i \varepsilon_{132} \tau_2 = -i \tau_2$$

$$\tau_2 \tau_3 = i \varepsilon_{231} \tau_1 = i \tau_1$$

$$\tau_1 \tau_1 = \mathbf{I}$$

$$\tau_2 \tau_2 = \mathbf{I}$$

$$\tau_3 \tau_3 = \mathbf{I}$$

$$\tau_2 \tau_1 = i \varepsilon_{213} \tau_3 = -i \tau_3$$

$$\tau_3 \tau_1 = i \varepsilon_{312} \tau_2 = i \tau_2$$

$$\tau_3 \tau_2 = i \varepsilon_{321} \tau_1 = -i \tau_1$$