NOTES QUANTIQUES

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$$i\gamma^{\mu}\partial_{\mu\psi} - m\psi = 0$$

Dirac

$$-\gamma_{v}\gamma^{\mu}\partial^{v}\partial_{\mu\psi} - im\gamma_{v}\partial^{v}\psi = 0 \qquad\qquad im\gamma_{v}\partial^{v}\psi = m^{2}\psi$$

$$\gamma_{v}\gamma^{\mu}\partial^{v}\partial_{\mu\psi} + m^{2}\psi = 0$$

$* \{\gamma^{\mu},\gamma^{\nu}\} = \gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}$ Clifford – Minkowski

$$\gamma_{v}\gamma^{\mu}\partial^{v}\partial_{\mu} = g_{\nu\sigma}\gamma^{\sigma}\gamma^{\mu}\partial^{\nu}\partial_{\mu} = g_{\nu\sigma}\frac{(\gamma^{\mu\sigma} + \gamma^{\sigma}\gamma^{\mu})}{2}\partial^{\nu}\partial_{\mu} = g_{\nu\sigma}\frac{2g^{\mu\sigma}}{2}\partial^{\nu}\partial_{\mu} = g_{\nu\sigma}\partial^{\nu}\partial_{\mu} = \delta_{v}^{\mu}\partial^{\nu}\partial_{\mu}$$

$$\gamma_{v}\gamma^{\mu}\partial^{v}\partial_{\mu\psi} + m^{2}\psi = \delta_{v}^{\mu}\partial^{v}\partial_{\mu}\psi + m^{2}\psi = \partial^{\mu}\partial_{\mu\psi} + m^{2}\psi = 0$$

Klein – Gordon

Notation: $\gamma^{\nu}\gamma^{\mu} = \frac{1}{2}\gamma^{\nu}\gamma^{\mu} + \frac{1}{2}\gamma^{\nu}\gamma^{\mu} - \frac{1}{2}\gamma^{\mu}\gamma^{\nu} + \frac{1}{2}\gamma^{\mu}\gamma^{\nu}$

$$\gamma^{\nu}\gamma^{\mu} = \frac{1}{2}(\gamma^{\nu}\mu^{\nu} - \gamma^{\mu}\gamma^{\nu}) + \frac{1}{2}(\gamma^{\nu}\gamma^{\mu} + \gamma^{\mu}\gamma^{\nu})$$

Lie – Clifford: $\gamma^{\nu}\gamma^{\mu} = \gamma^{[\nu}\gamma^{\mu]} + \gamma^{\{\nu}\gamma^{\mu\}} = \frac{1}{2}[\gamma^{\nu},\gamma^{\mu}] + g^{\mu\nu}$

$$\gamma^{\nu}\gamma^{\mu} = -2i\,S^{\nu\mu} + g^{\mu\nu}$$

Einstein: $E^{2} = p^{2}c^{2} + m^{2}c^{4}$

Schrodinger: $E\psi = -\frac{\hbar^{2}}{2m}\frac{\partial^{2}\psi}{\partial x^{2}} + V\psi$

Substitutions: $E \to i\hbar\frac{\partial}{\partial t}$, $\quad p \to -i\hbar\frac{\partial}{\partial x}$, $\quad \vec{p} \to -i\hbar\nabla$

$$E^{2} \to -\hbar^{2}\frac{\partial^{2}}{\partial t^{2}}, \quad p^{2} = -\hbar^{2}\nabla^{2}$$

$$E + S \Longrightarrow \quad -\hbar^{2}\frac{\partial^{2}}{\partial t^{2}} = \hbar^{2}c^{2}\nabla^{2} + m^{2}c^{4}$$

$$\hbar^2 \frac{\partial^2 \varphi}{\partial t^2} - \hbar^2 c^2 \nabla^2 \varphi + m^2 c^4 \varphi = 0$$

$\hbar = c = 1$

$$\frac{\partial^2 \varphi}{\partial t^2} - \nabla^2 \varphi + m^2 \varphi = 0$$

D'Alembert – Minkowski: $\square = \frac{\partial^2}{\partial t^2} - \nabla^2$

Klein – Gordon: $(\square + m^2)\varphi = 0$

Dirac: $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

$\sigma_{\mu\nu}\sigma^{\mu\nu} = 12$, $\sigma_{\mu\nu}\gamma_\kappa\sigma^{\mu\nu} = 0$, $\sigma_{\mu\nu}\gamma^5\sigma^{\mu\nu} = 12\gamma^5$

$\sigma_{\mu\nu}\gamma_k\sigma^{\mu\nu} = 0$, $\sigma_{\mu\nu}\sigma^{\kappa\lambda}\sigma^{\mu\nu} = -4\sigma^{\kappa\lambda}$

Lagrange: $\hat{\mathcal{L}} = \frac{1}{2}\partial_\mu\hat{\varphi}_1\partial^\mu\hat{\varphi}_1 - \frac{1}{2}M^2\hat{\varphi}_1^2 + \frac{1}{2}\partial_\mu\hat{\varphi}_2\partial^\mu\hat{\varphi}_2 - \frac{1}{2}M^2\hat{\varphi}_2^2$

Invariance: $\begin{cases}\hat{\varphi}_1' = (\cos\alpha)\hat{\varphi}_1 - (\sin\alpha)\hat{\varphi}_2 \\ \hat{\varphi}_2' = (\sin\alpha)\hat{\varphi}_1 + (\cos\alpha)\hat{\varphi}_2\end{cases}$ $\hat{\mathcal{L}}(\hat{\varphi}_1', \hat{\varphi}_2') = \hat{\mathcal{L}}(\hat{\varphi}_1, \hat{\varphi}_2)$

Voisinage 0: $\hat{\varphi}_1' = \hat{\varphi}_1 - \varepsilon\hat{\varphi}_2$, $\hat{\varphi}_2' = \hat{\varphi}_2 + \varepsilon\hat{\varphi}_1$

$\delta\hat{\varphi}_1 \equiv \hat{\varphi}_1' - \hat{\varphi}_1 = -\varepsilon\hat{\varphi}_2$, $\delta\hat{\varphi}_2 \equiv \hat{\varphi}_2' - \hat{\varphi}_2 = +\varepsilon\hat{\varphi}_1$

$\hat{\mathcal{L}}$ invariant $\Longrightarrow$ $\delta\hat{\mathcal{L}} = 0$

$$0 = \delta\hat{\mathcal{L}} = \frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_1)}\delta(\partial_\mu\hat{\varphi}_1) + \frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_2)}\delta(\partial_\mu\hat{\varphi}_2) + \frac{\partial\hat{\mathcal{L}}}{\partial\hat{\varphi}_1}\delta\hat{\varphi}_1 + \frac{\partial\hat{\mathcal{L}}}{\partial\hat{\varphi}_2}\delta\hat{\varphi}_2$$

$$0 = \frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_1)}\delta(\partial_\mu\hat{\varphi}_1) + \frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_2)}\delta(\partial_\mu\hat{\varphi}_2) + \left[\partial_\mu\left(\frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_1)}\right)\right]\delta\hat{\varphi}_1 + \left[\partial_\mu\left(\frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_2)}\right)\right]\delta\hat{\varphi}_2$$

$$\delta(\partial_\mu\hat{\varphi}_i) = \partial_\mu(\delta\hat{\varphi}_i)$$

$$\partial_\mu\left[\frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_1)}\delta\hat{\varphi}_1 + \frac{\partial\hat{\mathcal{L}}}{\partial(\partial_\mu\hat{\varphi}_2)}\delta\hat{\varphi}_2\right] = 0$$

$$\varepsilon\partial_\mu[(\partial^\mu\hat{\varphi}_2)\hat{\varphi}_1 - (\partial^\mu\hat{\varphi}_1)\hat{\varphi}_2] = 0$$

$$\hat{N}_\varphi^\mu = \hat{\varphi}_1\partial^\mu\hat{\varphi}_2 - \hat{\varphi}_2\partial^\mu\hat{\varphi}_1$$

Noether: $\partial_\mu\hat{N}_\varphi^\mu = 0$