Transformations infinitésimales, SU(2) et SU(3)

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$$F \longrightarrow F + dF = F + \sum_{i=1}^{N} \frac{\partial F}{\partial x_i} dx_i = F + \sum_{i=1}^{N} \left[\sum_{\nu=1}^{\nu} \frac{\partial f_i}{\partial a_\nu} da_\nu \right] \frac{\partial F}{\partial x_i}$$

$$F + dF = \{1 - \textstyle\sum_{\nu=1}^{\nu} da_\nu i \hat{x}_\nu\} F \qquad \hat{x}_\nu \equiv i \textstyle\sum_{i=1}^{N} \frac{\partial f_i}{\partial a_\nu} \frac{\partial}{\partial x_i}$$

SU(2)

$$\begin{pmatrix} q_1' \\ q_2' \end{pmatrix} = \left(1 + i\varepsilon\frac{\tau}{2}\right) \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$dq_1 = \frac{i\varepsilon_3}{2} q_1 + \left(\frac{\varepsilon_1}{2} + \frac{\varepsilon_2}{2}\right) q_2$$

$$dq_2 = -\frac{i\varepsilon_3}{2} q_2 + \left(\frac{\varepsilon_1}{2} - \frac{\varepsilon_2}{2}\right) q_1$$

$$da_i \equiv \varepsilon_i$$

$$\frac{\partial f_1}{\partial a_1} = \frac{iq_2}{2}, \quad \frac{\partial f_1}{\partial a_2} = \frac{q_2}{2}, \frac{\partial f_1}{\partial a_3} = \frac{iq_1}{2}$$

$$\frac{\partial f_2}{\partial a} = \frac{iq_1}{2}, \frac{\partial f_2}{\partial a_2} = -\frac{q_1}{2}, \frac{\partial f_2}{\partial a_3} = -\frac{iq_2}{2}$$

$$\hat{x}_1' = -\frac{1}{2}\left\{q_2 \frac{\partial}{\partial q_1} + q_2 \frac{\partial}{\partial q_2}\right\}$$

$$\hat{x}_2' = \frac{i}{2}\left\{q_2 \frac{\partial}{\partial q_1} - q_1 \frac{\partial}{\partial q_2}\right\}$$

$$\hat{x}_3' = \frac{1}{2}\left\{-q_1 \frac{\partial}{\partial q_1} + q_2 \frac{\partial}{\partial q_2}\right\}$$

SU(3)

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}' = \left(1 + i\frac{1}{2}\eta\lambda\right) \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

Gell – Mann $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \end{pmatrix}$

Algebre SU(3) $[\hat{G}_a, \hat{G}_b] = i f_{abc} \hat{G}_c$

$$\begin{cases} f_{123} = 1, f_{147} = \frac{1}{2}, f_{156} = -\frac{1}{2}, f_{246} = \frac{1}{2}, f_{257} = \frac{1}{2} \\ f_{345} = \frac{1}{2}, f_{367} = -\frac{1}{2}, f_{458} = \frac{\sqrt{3}}{2}, f_{678} = \frac{\sqrt{3}}{2} \\ f_{ijk} = 0 \end{cases}$$