The Number of Orders with Thirteen Elements

CLAUDE CHAUNIER and NIK LYGEROS

Département de Mathématiques, Université de Lyon -1, 43, boulevard du 11 Novembre 1918. 69622 Villeurbanne Cedex, France and Singularité, 141 avenue de Saxe 69003 Lyon, France

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Abstract. The number of non-isomorphic posets on 13 elements is $P_{13} = 33,823,827,452*$. This extends out previous result P_{12} which constituted the greatest known value. A table enumerates the posets according to their number of relations.

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Introduction

In 1932, D. H. Lehmer [5] proved that $2^{257} - 1$ was composite - whereas Mersenne believed it was prime using an ordinary calculating-machine 2 hours a day for a year. As for us. we used nine Apollo workstations - in particular one 400, one 3500 and seven 2500 for 6 months in order to achieve our calculations, and a VAX for 2 months and a RISC System/6000 for 2 weeks in order to partially verify them, all of that with a background, low priority basis. And thus an exhaustive verification still has to be done. On January 30th, 1952, the electronic calculating-machine SWAC managed by R. M. Robinson gave Lehmer's result in 48 seconds. Soon, it may possibly be the same for us. Here we consider only pairwise non-isomorphic posets and note P_n (resp. P_n^r) the number of those posets having n elements (and r relations).

$0 \leqslant n \leqslant 6$, $P_n = 1; 1; 2; 5; 16; 63; 318;$		
$P_7 = 2\,045$	(1972)	J. Wright
$P_8 = 16\,999$	(1977)	S. K. Das
$P_9 = 183\,231$	(1984)	R. H. Möhring
$P_{10} = 2\,567\,284$	(1990)	J. C. Culberson and G. J. E. Rawlins
$P_{11} = 46\,749\,427$	(1990)	J. C. Culberson and G. J. E. Rawlins
$P_{12} = 1\,104\,891\,746$	(1991)	C. Chaunier and N. Lygerōs
$P_{13} = 33\,823\,827\,452$	(1992)	

Results

The last value follows [4] - origin of this work - and [1] - where our algorithm is described. The following table gives the precise results.

r	P_{13}^r	r	P_{13}^r	r	P_{13}^r	r	P_{13}^r
0	1	20	36 606 102	40	1 805 816 407	60	6 096 379
1	1	21	63 090 851	41	1 633 935 577	61	3 796 733
2	3	22	103 573 457	42	1 446 444 433	62	2 320 757
3	7	23	162 384 152	43	1 253 457 366	63	1 391 478
4	19	24	243 809 985	44	1 063 880 995	64	817 624
5	47	25	351 390 204	45	884 825 225	65	470 396
6	133	26	487 237 576	46	721 452 090	66	264 558
7	354	27	651 206 672	47	576 924 933	67	145 258
8	1 014	28	840 404 152	48	452 654 555	68	77 647
9	2 874	29	1 048 785 819	49	348 576 046	69	40 260
10	8 305	30	1 267 416 540	50	263 545 083	70	20 165
11	23 513	31	1 484 925 018	51	195 684 307	71	9 660
12	65 215	32	1 688 672 630	52	142 728 742	72	4 391
13	173 481	33	1 865 878 896	53	102 283 393	73	1 862
14	441 249	34	2 005 172 954	54	72 028 601	74	714
15	1 062 532	35	2 097 659 160	55	49 849 120	75	241
16	2 419 194	36	2 138 021 170	56	33 906 587	76	66
17	5 194 267	37	2 124 818 344	57	22 666 616	77	12
18	10 529 510	38	2 060 635 454	58	14 891 283	78	1
19	20 169 973	39	1 951 423 800	59	9 613 263		

These results confirm R. Fraïssé's conjecture on unimodality. The values for $0 \leq r \leq 7$ agree with J.C. Culberson and G.J.E Rawlin's [2], and the values for $66 \leq r \leq 78$ are confirmed by M. Erné's formulae [3].

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