Υπολογισμός συντελεστών Clebsch-Gordan

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$$\left\langle j_1 j_2; m_1 m_2 \middle| j_1 j_2; j\, m \right\rangle$$

$$= \delta_{m,m_1+m_2} \sqrt{\frac{(2j+1)(j+j_1-j_2)!(j-j_1+j_2)!(j_1+j_2-j)!}{(j_1+j_2+j+1)!}} \times$$

$$\sqrt{(j+m)!(j-m)!(j_1-m_1)!(j_1+m_1)!(j_2-m_2)!(j_2+m_2)!} \times$$

$$\sum_k \frac{(-1)^k}{k!(j_1+j_2-j-k)!(j_1-m_1-k)!(j_2+m_2-k)!(j-j_2+m_1+k)!(j-j_1+m_2+k)}$$

Έστω

$$j_1 = j_2 = 1, \quad j = 2$$

Για $m = 2 \Rightarrow m_1 = m_2 = 1$

$$\left\langle 11;11 \middle| 11;22 \right\rangle = \delta_{2,2} \sqrt{\frac{(\cancel{5})2!2!0!}{\cancel{5}!}} \times \sqrt{4!0!0!2!0!2!} \times$$

$$\sum_k \frac{(-1)^k}{k!(-\cancel{k})!(-\cancel{k})!(2-k)!(2+k)!k!}$$

$$\left\langle 11;11 \middle| 11;22 \right\rangle = 4 * \sum_{k=2} \frac{(-1)^k}{k!k!(2-k)!(2+k)!} = \frac{4}{4} = 1$$

Έστω $j_1 = j_2 = 1$, $j = 2$

Για $m = 1 \Rightarrow m_1 = 1, m_2 = 0$ ή $m_1 = 0, m_2 = 1$

$$\left\langle 11;10 \middle| 11;21 \right\rangle = \delta_{1,1} \sqrt{\frac{(5)2!2!0!}{5!}} \times \sqrt{3!1!0!2!1!1!} \times$$

$$\sum_k \frac{(-1)^k}{k!(-\cancel{k})!(-\cancel{k})!(1-k)!(2+k)!(1+k)!}$$

$$\langle 11;10|11;21\rangle = \sqrt{2}\sum_{k=0}\frac{(-1)^k}{k!(1-k)!(2+k)!(1+k)!} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Έστω για $j_1 = j_2 = 1$, $j = 1$

Για $m = 1 \Rightarrow m_1 = 1, m_2 = 0$

$$\langle 11;10|11;21\rangle = \delta_{1,1}\sqrt{\frac{(3)1!1!1!}{4!}} \times \sqrt{2!1!0!2!1!1!} \times$$

$$\sum_k \frac{(-1)^k}{k!(-k)!(-k)!(1-k)!(1+k)!k!} = \frac{1}{\sqrt{2}} - 1 = \frac{1}{\sqrt{2}}$$

Έστω $j_1 = j_2 = 1$, $j = 1$

Για $m = 1 \Rightarrow m_1 = 0, m_2 = 1$

$$\langle 11;01|11;11\rangle = \delta_{1,1}\sqrt{\frac{(3)1!1!1!}{4!}} \times \sqrt{2!1!0!2!1!1!} \times$$

$$\sum_k \frac{(-1)^k}{k!(1-k)!(1-k)!(2-k)!k!(-1+k)!}$$

$$\langle 11;01|11;11\rangle = \frac{1}{\sqrt{2}} * \sum_{k=1}\frac{(-1)^k}{(2-k)!k!k!} = \frac{1}{\sqrt{2}}$$