

```

> #Brinkmann metric
s:=2*D(U)*D(V)+Dirac(u)*(Y^2-X^2)*(D(u))^2-(D(X))^2-(D(Y))^2;
s := 2 D(U) D(V) + Dirac(u) ( $Y^2 - X^2$ ) D(u) $^2 - D(X)^2 - D(Y)^2$  (1)
> a:=subs({U=u,V=v-1/2*Heaviside(u)*(1-u)*x^2+1/2*Heaviside(u)*(1+u)*y^2,X=(1-u*Heaviside(u))*x,Y=(1+u*Heaviside(u))*y},s);
a := 2 D(u) D $\left(v - \frac{1}{2} \text{Heaviside}(u) (1 - u) x^2 + \frac{1}{2} \text{Heaviside}(u) (1 + u) y^2\right)$  (2)
+ Dirac(u) ((1 + u Heaviside(u)) $^2 y^2 - (1 - u \text{Heaviside}(u)) $^2 x^2) D(u) $^2 - D((1 - u \text{Heaviside}(u)) x)^2 - D((1 + u \text{Heaviside}(u)) y)^2$ 
> b:=expand(a);
b := 2 D(x) $^2 u \text{Heaviside}(u) + D(u) D(\text{Heaviside}(u)) x^2 u + D(u) D(\text{Heaviside}(u)) y^2 u$  (3)
+ 2 x u D(Heaviside(u)) D(x) - 2 y u D(Heaviside(u)) D(y)
+ 2 D(u) Heaviside(u) D(x) x u + 2 D(u) Heaviside(u) D(y) y u
+ 2 Dirac(u) D(u) $^2 \text{Heaviside}(u) y^2 u + \text{Dirac}(u) D(u) $^2 y^2 u^2 \text{Heaviside}(u)^2$ 
+ 2 Dirac(u) D(u) $^2 \text{Heaviside}(u) x^2 u - \text{Dirac}(u) D(u) $^2 x^2 u^2 \text{Heaviside}(u)^2$ 
- 2 x $^2 D(u) \text{Heaviside}(u) u D(\text{Heaviside}(u)) - 2 x D(u) \text{Heaviside}(u)^2 D(x) u$ 
- 2 x u $^2 D(\text{Heaviside}(u)) D(x) \text{Heaviside}(u)$ 
- 2 y $^2 D(u) \text{Heaviside}(u) u D(\text{Heaviside}(u)) - 2 y D(u) \text{Heaviside}(u)^2 D(y) u$ 
- 2 y u $^2 D(\text{Heaviside}(u)) D(y) \text{Heaviside}(u) - D(u) D(\text{Heaviside}(u)) x^2$ 
+ Heaviside(u) D(u) $^2 x^2 - y^2 D(u)^2 \text{Heaviside}(u)^2 + D(u) D(\text{Heaviside}(u)) y^2$ 
+ Heaviside(u) D(u) $^2 y^2 + \text{Dirac}(u) D(u)^2 y^2 - \text{Dirac}(u) D(u)^2 x^2$ 
- x $^2 D(u)^2 \text{Heaviside}(u)^2 - x^2 u^2 D(\text{Heaviside}(u))^2 - D(x)^2 u^2 \text{Heaviside}(u)^2$ 
- y $^2 u^2 D(\text{Heaviside}(u))^2 - 2 D(y)^2 u \text{Heaviside}(u) - D(y)^2 u^2 \text{Heaviside}(u)^2$ 
+ 2 D(u) D(v) - D(x) $^2 - D(y)^2$ 
> c:=simplify(b);
c := 2 D(x) $^2 u \text{Heaviside}(u) - 2 D(y)^2 u \text{Heaviside}(u) + 2 D(u) D(v) - D(x)^2 - D(y)^2$  (4)
- D(x) $^2 u^2 \text{Heaviside}(u) - D(y)^2 u^2 \text{Heaviside}(u)$ 
> factor(c);
2 D(x) $^2 u \text{Heaviside}(u) - 2 D(y)^2 u \text{Heaviside}(u) + 2 D(u) D(v) - D(x)^2 - D(y)^2$  (5)
- D(x) $^2 u^2 \text{Heaviside}(u) - D(y)^2 u^2 \text{Heaviside}(u)$ 
> #Specific package
> with(DifferentialGeometry);
[&minus, &mult, &plus, &tensor, &wedge, Annihilator, ApplyTransformation, ChangeFrame,
ComplementaryBasis, ComposeTransformations, DGbasis, DGsetup, DGzip,
DeRhamHomotopy, DualBasis, ExteriorDerivative, Flow, FrameData, GetComponents,
GroupActions, Hook, InfinitesimalTransformation, IntegrateForm, IntersectSubspaces,
InverseTransformation, JetCalculus, Library, LieAlgebras, LieBracket, LieDerivative,
Preferences, Pullback, PullbackVector, Pushforward, RemoveFrame, Tensor, Tools,
Transformation, evalDG] (6)$$$$ 
```

```
> with(tensor); #no with(Tensor)
[Christoffel1, Christoffel2, Einstein, Jacobian, Killing_eqns, Levi_Civita, Lie_diff, Ricci,
 Ricciscalar, Riemann, RiemannF, Weyl, act, antisymmetrize, change_basis, commutator,
 compare, conj, connexF, contract, convertNP, cov_diff, create, d1metric, d2metric,
 directional_diff, displayGR, display_allGR, dual, entermetric, exterior_diff, exterior_prod,
 frame, geodesic_eqns, get_char, get_compts, get_rank, init, invars, invert, lin_com, lower,
 npcurve, npspin, partial_diff, permute_indices, petrov, prod, raise, symmetrize, tensorsGR,
 transform]
```

```
> coord:=[u,v,x,y]:
> g_compts:=array(symmetric, sparse, 1..4, 1..4):
> g_compts[1,2]:=1:
> g_compts[2,1]:=1:
> g_compts[3,3]:=- (1-u*Heaviside(u))^2:
> g_compts[4,4]:=- (1+u*Heaviside(u))^2:
> g:=create([-1,-1],eval(g_compts)); #metric
```

$$g := \text{table} \left(\begin{array}{l} \text{index_char} = [-1, -1], \text{compts} \\ \end{array} \right) \quad (8)$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -(1 - u \text{Heaviside}(u))^2 & 0 \\ 0 & 0 & 0 & -(1 + u \text{Heaviside}(u))^2 \end{bmatrix}$$

```
> ginv:=invert(g, `detg`);
```

$$\text{ginv} := \text{table} \left(\begin{array}{l} \text{index_char} = [1, 1], \text{compts} \\ \end{array} \right) \quad (9)$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{(-1 + u \text{Heaviside}(u))^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{(1 + u \text{Heaviside}(u))^2} \end{bmatrix}$$

```
> D1g:=d1metric(g, coord); #First derivative
```

$$D1g := \text{table}([\text{index_char} = [-1, -1, -1], \text{compts} = \text{ARRAY}(cfl, [1..4, 1..4, 1..4], [(1, 1, 1)])]) \quad (10)$$

```

=0, (1, 1, 2) =0, (1, 1, 3) =0, (1, 1, 4) =0, (1, 2, 1) =0, (1, 2, 2) =0, (1, 2, 3) =0, (1,
2, 4) =0, (1, 3, 1) =0, (1, 3, 2) =0, (1, 3, 3) =0, (1, 3, 4) =0, (1, 4, 1) =0, (1, 4, 2) =0,
(1, 4, 3) =0, (1, 4, 4) =0, (2, 1, 1) =0, (2, 1, 2) =0, (2, 1, 3) =0, (2, 1, 4) =0, (2, 2, 1)
=0, (2, 2, 2) =0, (2, 2, 3) =0, (2, 2, 4) =0, (2, 3, 1) =0, (2, 3, 2) =0, (2, 3, 3) =0, (2,
3, 4) =0, (2, 4, 1) =0, (2, 4, 2) =0, (2, 4, 3) =0, (2, 4, 4) =0, (3, 1, 1) =0, (3, 1, 2) =0,
(3, 1, 3) =0, (3, 1, 4) =0, (3, 2, 1) =0, (3, 2, 2) =0, (3, 2, 3) =0, (3, 2, 4) =0, (3, 3, 1)
=-2 (-1 + u Heaviside(u)) (Heaviside(u) + u Dirac(u)), (3, 3, 2) =0, (3, 3, 3) =0,
(3, 3, 4) =0, (3, 4, 1) =0, (3, 4, 2) =0, (3, 4, 3) =0, (3, 4, 4) =0, (4, 1, 1) =0, (4, 1, 2)
=0, (4, 1, 3) =0, (4, 1, 4) =0, (4, 2, 1) =0, (4, 2, 2) =0, (4, 2, 3) =0, (4, 2, 4) =0, (4,
3, 1) =0, (4, 3, 2) =0, (4, 3, 3) =0, (4, 3, 4) =0, (4, 4, 1) =-2 (1
+ u Heaviside(u)) (Heaviside(u) + u Dirac(u)), (4, 4, 2) =0, (4, 4, 3) =0, (4, 4, 4)
=0]]])

```

> Cf1:=Christoffell(D1g); #Christoffel symbols

```

Cf1:=table([index_char=[-1, -1, -1], compts=ARRAY(cfl, [1..4, 1..4, 1..4], [(1, 1, 1)
=0, (1, 1, 2) =0, (1, 1, 3) =0, (1, 1, 4) =0, (1, 2, 1) =0, (1, 2, 2) =0, (1, 2, 3) =0, (1,
2, 4) =0, (1, 3, 1) =0, (1, 3, 2) =0, (1, 3, 3) =-(-1 + u Heaviside(u)) (Heaviside(u)
+ u Dirac(u)), (1, 3, 4) =0, (1, 4, 1) =0, (1, 4, 2) =0, (1, 4, 3) =0, (1, 4, 4) =-(1
+ u Heaviside(u)) (Heaviside(u) + u Dirac(u)), (2, 1, 1) =0, (2, 1, 2) =0, (2, 1, 3)
=0, (2, 1, 4) =0, (2, 2, 1) =0, (2, 2, 2) =0, (2, 2, 3) =0, (2, 2, 4) =0, (2, 3, 1) =0, (2,
3, 2) =0, (2, 3, 3) =0, (2, 3, 4) =0, (2, 4, 1) =0, (2, 4, 2) =0, (2, 4, 3) =0, (2, 4, 4) =0,
(3, 1, 1) =0, (3, 1, 2) =0, (3, 1, 3) =-(-1 + u Heaviside(u)) (Heaviside(u)
+ u Dirac(u)), (3, 1, 4) =0, (3, 2, 1) =0, (3, 2, 2) =0, (3, 2, 3) =0, (3, 2, 4) =0, (3, 3,
1) =(-1 + u Heaviside(u)) (Heaviside(u) + u Dirac(u)), (3, 3, 2) =0, (3, 3, 3) =0, (3,
3, 4) =0, (3, 4, 1) =0, (3, 4, 2) =0, (3, 4, 3) =0, (3, 4, 4) =0, (4, 1, 1) =0, (4, 1, 2) =0,
(4, 1, 3) =0, (4, 1, 4) =- (1 + u Heaviside(u)) (Heaviside(u) + u Dirac(u)), (4, 2, 1)
=0, (4, 2, 2) =0, (4, 2, 3) =0, (4, 2, 4) =0, (4, 3, 1) =0, (4, 3, 2) =0, (4, 3, 3) =0, (4,
3, 4) =0, (4, 4, 1) =(1 + u Heaviside(u)) (Heaviside(u) + u Dirac(u)), (4, 4, 2) =0,
(4, 4, 3) =0, (4, 4, 4) =0]]])

```

> D2g:=d2metric(D1g, coord); #Second derivative

```

D2g:=table([index_char=[-1, -1, -1, -1], compts=ARRAY(d2met, [1..4, 1..4, 1..4, 1
..4], [(1, 1, 1, 1) =0, (1, 1, 1, 2) =0, (1, 1, 1, 3) =0, (1, 1, 1, 4) =0, (1, 1, 2, 1) =0, (1,
1, 2, 2) =0, (1, 1, 2, 3) =0, (1, 1, 2, 4) =0, (1, 1, 3, 1) =0, (1, 1, 3, 2) =0, (1, 1, 3, 3)
=0, (1, 1, 3, 4) =0, (1, 1, 4, 1) =0, (1, 1, 4, 2) =0, (1, 1, 4, 3) =0, (1, 1, 4, 4) =0, (1, 2,
1, 1) =0, (1, 2, 1, 2) =0, (1, 2, 1, 3) =0, (1, 2, 1, 4) =0, (1, 2, 2, 1) =0, (1, 2, 2, 2) =0,
(1, 2, 2, 3) =0, (1, 2, 2, 4) =0, (1, 2, 3, 1) =0, (1, 2, 3, 2) =0, (1, 2, 3, 3) =0, (1, 2, 3, 4)
=0, (1, 2, 4, 1) =0, (1, 2, 4, 2) =0, (1, 2, 4, 3) =0, (1, 2, 4, 4) =0, (1, 3, 1, 1) =0, (1, 3,
1, 2) =0, (1, 3, 1, 3) =0, (1, 3, 1, 4) =0, (1, 3, 2, 1) =0, (1, 3, 2, 2) =0, (1, 3, 2, 3) =0,
(1, 3, 2, 4) =0, (1, 3, 3, 1) =0, (1, 3, 3, 2) =0, (1, 3, 3, 3) =0, (1, 3, 3, 4) =0, (1, 3, 4, 1)
=0, (1, 3, 4, 2) =0, (1, 3, 4, 3) =0, (1, 3, 4, 4) =0, (1, 4, 1, 1) =0, (1, 4, 1, 2) =0, (1, 4,

```

$$\begin{aligned}
& 1, 3) = 0, (1, 4, 1, 4) = 0, (1, 4, 2, 1) = 0, (1, 4, 2, 2) = 0, (1, 4, 2, 3) = 0, (1, 4, 2, 4) = 0, \\
& (1, 4, 3, 1) = 0, (1, 4, 3, 2) = 0, (1, 4, 3, 3) = 0, (1, 4, 3, 4) = 0, (1, 4, 4, 1) = 0, (1, 4, 4, 2) \\
& = 0, (1, 4, 4, 3) = 0, (1, 4, 4, 4) = 0, (2, 1, 1, 1) = 0, (2, 1, 1, 2) = 0, (2, 1, 1, 3) = 0, (2, 1, \\
& 1, 4) = 0, (2, 1, 2, 1) = 0, (2, 1, 2, 2) = 0, (2, 1, 2, 3) = 0, (2, 1, 2, 4) = 0, (2, 1, 3, 1) = 0, \\
& (2, 1, 3, 2) = 0, (2, 1, 3, 3) = 0, (2, 1, 3, 4) = 0, (2, 1, 4, 1) = 0, (2, 1, 4, 2) = 0, (2, 1, 4, 3) \\
& = 0, (2, 1, 4, 4) = 0, (2, 2, 1, 1) = 0, (2, 2, 1, 2) = 0, (2, 2, 1, 3) = 0, (2, 2, 1, 4) = 0, (2, 2, \\
& 2, 1) = 0, (2, 2, 2, 2) = 0, (2, 2, 2, 3) = 0, (2, 2, 2, 4) = 0, (2, 2, 3, 1) = 0, (2, 2, 3, 2) = 0, \\
& (2, 2, 3, 3) = 0, (2, 2, 3, 4) = 0, (2, 2, 4, 1) = 0, (2, 2, 4, 2) = 0, (2, 2, 4, 3) = 0, (2, 2, 4, 4) \\
& = 0, (2, 3, 1, 1) = 0, (2, 3, 1, 2) = 0, (2, 3, 1, 3) = 0, (2, 3, 1, 4) = 0, (2, 3, 2, 1) = 0, (2, 3, \\
& 2, 2) = 0, (2, 3, 2, 3) = 0, (2, 3, 2, 4) = 0, (2, 3, 3, 1) = 0, (2, 3, 3, 2) = 0, (2, 3, 3, 3) = 0, \\
& (2, 3, 3, 4) = 0, (2, 3, 4, 1) = 0, (2, 3, 4, 2) = 0, (2, 3, 4, 3) = 0, (2, 3, 4, 4) = 0, (2, 4, 1, 1) \\
& = 0, (2, 4, 1, 2) = 0, (2, 4, 1, 3) = 0, (2, 4, 1, 4) = 0, (2, 4, 2, 1) = 0, (2, 4, 2, 2) = 0, (2, 4, \\
& 2, 3) = 0, (2, 4, 2, 4) = 0, (2, 4, 3, 1) = 0, (2, 4, 3, 2) = 0, (2, 4, 3, 3) = 0, (2, 4, 3, 4) = 0, \\
& (2, 4, 4, 1) = 0, (2, 4, 4, 2) = 0, (2, 4, 4, 3) = 0, (2, 4, 4, 4) = 0, (3, 1, 1, 1) = 0, (3, 1, 1, 2) \\
& = 0, (3, 1, 1, 3) = 0, (3, 1, 1, 4) = 0, (3, 1, 2, 1) = 0, (3, 1, 2, 2) = 0, (3, 1, 2, 3) = 0, (3, 1, \\
& 2, 4) = 0, (3, 1, 3, 1) = 0, (3, 1, 3, 2) = 0, (3, 1, 3, 3) = 0, (3, 1, 3, 4) = 0, (3, 1, 4, 1) = 0, \\
& (3, 1, 4, 2) = 0, (3, 1, 4, 3) = 0, (3, 1, 4, 4) = 0, (3, 2, 1, 1) = 0, (3, 2, 1, 2) = 0, (3, 2, 1, 3) \\
& = 0, (3, 2, 1, 4) = 0, (3, 2, 2, 1) = 0, (3, 2, 2, 2) = 0, (3, 2, 2, 3) = 0, (3, 2, 2, 4) = 0, (3, 2, \\
& 3, 1) = 0, (3, 2, 3, 2) = 0, (3, 2, 3, 3) = 0, (3, 2, 3, 4) = 0, (3, 2, 4, 1) = 0, (3, 2, 4, 2) = 0, \\
& (3, 2, 4, 3) = 0, (3, 2, 4, 4) = 0, (3, 3, 1, 1) = -2 \text{Heaviside}(u)^2 \\
& - 8 \text{Heaviside}(u) u \text{Dirac}(u) - 2 u^2 \text{Dirac}(u)^2 + 4 \text{Dirac}(u) + 2 u \text{Dirac}(1, u) \\
& - 2 u^2 \text{Heaviside}(u) \text{Dirac}(1, u), (3, 3, 1, 2) = 0, (3, 3, 1, 3) = 0, (3, 3, 1, 4) = 0, (3, 3, 2, \\
& 1) = 0, (3, 3, 2, 2) = 0, (3, 3, 2, 3) = 0, (3, 3, 2, 4) = 0, (3, 3, 3, 1) = 0, (3, 3, 3, 2) = 0, (3, \\
& 3, 3, 3) = 0, (3, 3, 3, 4) = 0, (3, 3, 4, 1) = 0, (3, 3, 4, 2) = 0, (3, 3, 4, 3) = 0, (3, 3, 4, 4) \\
& = 0, (3, 4, 1, 1) = 0, (3, 4, 1, 2) = 0, (3, 4, 1, 3) = 0, (3, 4, 1, 4) = 0, (3, 4, 2, 1) = 0, (3, 4, \\
& 2, 2) = 0, (3, 4, 2, 3) = 0, (3, 4, 2, 4) = 0, (3, 4, 3, 1) = 0, (3, 4, 3, 2) = 0, (3, 4, 3, 3) = 0, \\
& (3, 4, 3, 4) = 0, (3, 4, 4, 1) = 0, (3, 4, 4, 2) = 0, (3, 4, 4, 3) = 0, (3, 4, 4, 4) = 0, (4, 1, 1, 1) \\
& = 0, (4, 1, 1, 2) = 0, (4, 1, 1, 3) = 0, (4, 1, 1, 4) = 0, (4, 1, 2, 1) = 0, (4, 1, 2, 2) = 0, (4, 1, \\
& 2, 3) = 0, (4, 1, 2, 4) = 0, (4, 1, 3, 1) = 0, (4, 1, 3, 2) = 0, (4, 1, 3, 3) = 0, (4, 1, 3, 4) = 0, \\
& (4, 1, 4, 1) = 0, (4, 1, 4, 2) = 0, (4, 1, 4, 3) = 0, (4, 1, 4, 4) = 0, (4, 2, 1, 1) = 0, (4, 2, 1, 2) \\
& = 0, (4, 2, 1, 3) = 0, (4, 2, 1, 4) = 0, (4, 2, 2, 1) = 0, (4, 2, 2, 2) = 0, (4, 2, 2, 3) = 0, (4, 2, \\
& 2, 4) = 0, (4, 2, 3, 1) = 0, (4, 2, 3, 2) = 0, (4, 2, 3, 3) = 0, (4, 2, 3, 4) = 0, (4, 2, 4, 1) = 0, \\
& (4, 2, 4, 2) = 0, (4, 2, 4, 3) = 0, (4, 2, 4, 4) = 0, (4, 3, 1, 1) = 0, (4, 3, 1, 2) = 0, (4, 3, 1, 3) \\
& = 0, (4, 3, 1, 4) = 0, (4, 3, 2, 1) = 0, (4, 3, 2, 2) = 0, (4, 3, 2, 3) = 0, (4, 3, 2, 4) = 0, (4, 3, \\
& 3, 1) = 0, (4, 3, 3, 2) = 0, (4, 3, 3, 3) = 0, (4, 3, 3, 4) = 0, (4, 3, 4, 1) = 0, (4, 3, 4, 2) = 0, \\
& (4, 3, 4, 3) = 0, (4, 3, 4, 4) = 0, (4, 4, 1, 1) = -2 \text{Heaviside}(u)^2
\end{aligned}$$

$$\begin{aligned}
& -8 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) - 2 u^2 \operatorname{Dirac}(u)^2 - 4 \operatorname{Dirac}(u) - 2 u \operatorname{Dirac}(1, u) \\
& - 2 u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (4, 4, 1, 2) = 0, (4, 4, 1, 3) = 0, (4, 4, 1, 4) = 0, (4, 4, 2, \\
& 1) = 0, (4, 4, 2, 2) = 0, (4, 4, 2, 3) = 0, (4, 4, 2, 4) = 0, (4, 4, 3, 1) = 0, (4, 4, 3, 2) = 0, (4, \\
& 4, 3, 3) = 0, (4, 4, 3, 4) = 0, (4, 4, 4, 1) = 0, (4, 4, 4, 2) = 0, (4, 4, 4, 3) = 0, (4, 4, 4, 4) \\
& = 0])]
\end{aligned}$$

> **RIEMANN:=Riemann(ginv, D2g, Cf1); #Riemann Tensor**

$$\begin{aligned}
& RIEMANN := \text{table}([index_char = [-1, -1, -1, -1], compts = \text{ARRAY}(cov_riemann, [1..4, 1 \\
& ..4, 1..4, 1..4], [(1, 1, 1, 1) = 0, (1, 1, 1, 2) = 0, (1, 1, 1, 3) = 0, (1, 1, 1, 4) = 0, (1, 1, 2, \\
& 1) = 0, (1, 1, 2, 2) = 0, (1, 1, 2, 3) = 0, (1, 1, 2, 4) = 0, (1, 1, 3, 1) = 0, (1, 1, 3, 2) = 0, (1, \\
& 1, 3, 3) = 0, (1, 1, 3, 4) = 0, (1, 1, 4, 1) = 0, (1, 1, 4, 2) = 0, (1, 1, 4, 3) = 0, (1, 1, 4, 4) \\
& = 0, (1, 2, 1, 1) = 0, (1, 2, 1, 2) = 0, (1, 2, 1, 3) = 0, (1, 2, 1, 4) = 0, (1, 2, 2, 1) = 0, (1, 2, \\
& 2, 2) = 0, (1, 2, 2, 3) = 0, (1, 2, 2, 4) = 0, (1, 2, 3, 1) = 0, (1, 2, 3, 2) = 0, (1, 2, 3, 3) = 0, \\
& (1, 2, 3, 4) = 0, (1, 2, 4, 1) = 0, (1, 2, 4, 2) = 0, (1, 2, 4, 3) = 0, (1, 2, 4, 4) = 0, (1, 3, 1, 1) \\
& = 0, (1, 3, 1, 2) = 0, (1, 3, 1, 3) = -2 \operatorname{Dirac}(u) - u \operatorname{Dirac}(1, u) \\
& + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (1, 3, 1, 4) = 0, (1, 3, 2, 1) \\
& = 0, (1, 3, 2, 2) = 0, (1, 3, 2, 3) = 0, (1, 3, 2, 4) = 0, (1, 3, 3, 1) = 2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, \\
& u) - 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) - u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (1, 3, 3, 2) = 0, (1, 3, 3, \\
& 3) = 0, (1, 3, 3, 4) = 0, (1, 3, 4, 1) = 0, (1, 3, 4, 2) = 0, (1, 3, 4, 3) = 0, (1, 3, 4, 4) = 0, (1, \\
& 4, 1, 1) = 0, (1, 4, 1, 2) = 0, (1, 4, 1, 3) = 0, (1, 4, 1, 4) = 2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u) \\
& + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (1, 4, 2, 1) = 0, (1, 4, 2, 2) \\
& = 0, (1, 4, 2, 3) = 0, (1, 4, 2, 4) = 0, (1, 4, 3, 1) = 0, (1, 4, 3, 2) = 0, (1, 4, 3, 3) = 0, (1, 4, \\
& 3, 4) = 0, (1, 4, 4, 1) = -2 \operatorname{Dirac}(u) - u \operatorname{Dirac}(1, u) - 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
& - u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (1, 4, 4, 2) = 0, (1, 4, 4, 3) = 0, (1, 4, 4, 4) = 0, (2, 1, 1, 1) \\
& = 0, (2, 1, 1, 2) = 0, (2, 1, 1, 3) = 0, (2, 1, 1, 4) = 0, (2, 1, 2, 1) = 0, (2, 1, 2, 2) = 0, (2, 1, \\
& 2, 3) = 0, (2, 1, 2, 4) = 0, (2, 1, 3, 1) = 0, (2, 1, 3, 2) = 0, (2, 1, 3, 3) = 0, (2, 1, 3, 4) = 0, \\
& (2, 1, 4, 1) = 0, (2, 1, 4, 2) = 0, (2, 1, 4, 3) = 0, (2, 1, 4, 4) = 0, (2, 2, 1, 1) = 0, (2, 2, 1, 2) \\
& = 0, (2, 2, 1, 3) = 0, (2, 2, 1, 4) = 0, (2, 2, 2, 1) = 0, (2, 2, 2, 2) = 0, (2, 2, 2, 3) = 0, (2, 2, \\
& 2, 4) = 0, (2, 2, 3, 1) = 0, (2, 2, 3, 2) = 0, (2, 2, 3, 3) = 0, (2, 2, 3, 4) = 0, (2, 2, 4, 1) = 0, \\
& (2, 2, 4, 2) = 0, (2, 2, 4, 3) = 0, (2, 2, 4, 4) = 0, (2, 3, 1, 1) = 0, (2, 3, 1, 2) = 0, (2, 3, 1, 3) \\
& = 0, (2, 3, 1, 4) = 0, (2, 3, 2, 1) = 0, (2, 3, 2, 2) = 0, (2, 3, 2, 3) = 0, (2, 3, 2, 4) = 0, (2, 3, \\
& 3, 1) = 0, (2, 3, 3, 2) = 0, (2, 3, 3, 3) = 0, (2, 3, 3, 4) = 0, (2, 3, 4, 1) = 0, (2, 3, 4, 2) = 0, \\
& (2, 3, 4, 3) = 0, (2, 3, 4, 4) = 0, (2, 4, 1, 1) = 0, (2, 4, 1, 2) = 0, (2, 4, 1, 3) = 0, (2, 4, 1, 4) \\
& = 0, (2, 4, 2, 1) = 0, (2, 4, 2, 2) = 0, (2, 4, 2, 3) = 0, (2, 4, 2, 4) = 0, (2, 4, 3, 1) = 0, (2, 4, \\
& 3, 2) = 0, (2, 4, 3, 3) = 0, (2, 4, 3, 4) = 0, (2, 4, 4, 1) = 0, (2, 4, 4, 2) = 0, (2, 4, 4, 3) = 0, \\
& (2, 4, 4, 4) = 0, (3, 1, 1, 1) = 0, (3, 1, 1, 2) = 0, (3, 1, 1, 3) = 2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u) \\
& - 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) - u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (3, 1, 1, 4) = 0, (3, 1, 2, 1)
\end{aligned} \tag{13}$$

$$\begin{aligned}
&= 0, (3, 1, 2, 2) = 0, (3, 1, 2, 3) = 0, (3, 1, 2, 4) = 0, (3, 1, 3, 1) = -2 \operatorname{Dirac}(u) \\
&\quad - u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (3, 1, 3, 2) \\
&= 0, (3, 1, 3, 3) = 0, (3, 1, 3, 4) = 0, (3, 1, 4, 1) = 0, (3, 1, 4, 2) = 0, (3, 1, 4, 3) = 0, (3, 1, \\
&\quad 4, 4) = 0, (3, 2, 1, 1) = 0, (3, 2, 1, 2) = 0, (3, 2, 1, 3) = 0, (3, 2, 1, 4) = 0, (3, 2, 2, 1) = 0, \\
&\quad (3, 2, 2, 2) = 0, (3, 2, 2, 3) = 0, (3, 2, 2, 4) = 0, (3, 2, 3, 1) = 0, (3, 2, 3, 2) = 0, (3, 2, 3, 3) \\
&= 0, (3, 2, 3, 4) = 0, (3, 2, 4, 1) = 0, (3, 2, 4, 2) = 0, (3, 2, 4, 3) = 0, (3, 2, 4, 4) = 0, (3, 3, \\
&\quad 1, 1) = 0, (3, 3, 1, 2) = 0, (3, 3, 1, 3) = 0, (3, 3, 1, 4) = 0, (3, 3, 2, 1) = 0, (3, 3, 2, 2) = 0, \\
&\quad (3, 3, 2, 3) = 0, (3, 3, 2, 4) = 0, (3, 3, 3, 1) = 0, (3, 3, 3, 2) = 0, (3, 3, 3, 3) = 0, (3, 3, 3, 4) \\
&= 0, (3, 3, 4, 1) = 0, (3, 3, 4, 2) = 0, (3, 3, 4, 3) = 0, (3, 3, 4, 4) = 0, (3, 4, 1, 1) = 0, (3, 4, \\
&\quad 1, 2) = 0, (3, 4, 1, 3) = 0, (3, 4, 1, 4) = 0, (3, 4, 2, 1) = 0, (3, 4, 2, 2) = 0, (3, 4, 2, 3) = 0, \\
&\quad (3, 4, 2, 4) = 0, (3, 4, 3, 1) = 0, (3, 4, 3, 2) = 0, (3, 4, 3, 3) = 0, (3, 4, 3, 4) = 0, (3, 4, 4, 1) \\
&= 0, (3, 4, 4, 2) = 0, (3, 4, 4, 3) = 0, (3, 4, 4, 4) = 0, (4, 1, 1, 1) = 0, (4, 1, 1, 2) = 0, (4, 1, \\
&\quad 1, 3) = 0, (4, 1, 1, 4) = -2 \operatorname{Dirac}(u) - u \operatorname{Dirac}(1, u) - 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
&\quad - u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u), (4, 1, 2, 1) = 0, (4, 1, 2, 2) = 0, (4, 1, 2, 3) = 0, (4, 1, 2, 4) \\
&= 0, (4, 1, 3, 1) = 0, (4, 1, 3, 2) = 0, (4, 1, 3, 3) = 0, (4, 1, 3, 4) = 0, (4, 1, 4, 1) \\
&= 2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, \\
&\quad u), (4, 1, 4, 2) = 0, (4, 1, 4, 3) = 0, (4, 1, 4, 4) = 0, (4, 2, 1, 1) = 0, (4, 2, 1, 2) = 0, (4, 2, \\
&\quad 1, 3) = 0, (4, 2, 1, 4) = 0, (4, 2, 2, 1) = 0, (4, 2, 2, 2) = 0, (4, 2, 2, 3) = 0, (4, 2, 2, 4) = 0, \\
&\quad (4, 2, 3, 1) = 0, (4, 2, 3, 2) = 0, (4, 2, 3, 3) = 0, (4, 2, 3, 4) = 0, (4, 2, 4, 1) = 0, (4, 2, 4, 2) \\
&= 0, (4, 2, 4, 3) = 0, (4, 2, 4, 4) = 0, (4, 3, 1, 1) = 0, (4, 3, 1, 2) = 0, (4, 3, 1, 3) = 0, (4, 3, \\
&\quad 1, 4) = 0, (4, 3, 2, 1) = 0, (4, 3, 2, 2) = 0, (4, 3, 2, 3) = 0, (4, 3, 2, 4) = 0, (4, 3, 3, 1) = 0, \\
&\quad (4, 3, 3, 2) = 0, (4, 3, 3, 3) = 0, (4, 3, 3, 4) = 0, (4, 3, 4, 1) = 0, (4, 3, 4, 2) = 0, (4, 3, 4, 3) \\
&= 0, (4, 3, 4, 4) = 0, (4, 4, 1, 1) = 0, (4, 4, 1, 2) = 0, (4, 4, 1, 3) = 0, (4, 4, 1, 4) = 0, (4, 4, \\
&\quad 2, 1) = 0, (4, 4, 2, 2) = 0, (4, 4, 2, 3) = 0, (4, 4, 2, 4) = 0, (4, 4, 3, 1) = 0, (4, 4, 3, 2) = 0, \\
&\quad (4, 4, 3, 3) = 0, (4, 4, 3, 4) = 0, (4, 4, 4, 1) = 0, (4, 4, 4, 2) = 0, (4, 4, 4, 3) = 0, (4, 4, 4, 4) \\
&= 0])]
\end{aligned}$$

> **RICCI:=Ricci(ginv, RIEMANN); #Ricci Tensor**

$$\begin{aligned}
RICCI &:= \text{table} \left(\left[\begin{array}{c|ccc} & & & \\ \hline \text{index_char} & [-1, -1], \text{compts} & & \\ & & & \end{array} \right] \right) \tag{14} \\
&= \left(\begin{array}{cc|ccc} \frac{2 \operatorname{Heaviside}(u) u (2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u))}{(1 + u \operatorname{Heaviside}(u)) (-1 + u \operatorname{Heaviside}(u))} & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 \\ 0 & & 0 & 0 & 0 \end{array} \right)
\end{aligned}$$

> **RICCI_SCALAR:=Ricccisclar(ginv, RICCI);**
 $RICCI_SCALAR := \text{table}([index_char = [], compts = 0])$ (15)

> **WEYL:=Weyl(g, RIEMANN, RICCI, RICCI_SCALAR);**

$WEYL := \text{table}\left(\left[index_char = [-1, -1, -1, -1], compts = \text{ARRAY}\left(cov_riemann, [1..4, 1..4, 1..4, 1..4], \left[\begin{array}{l} (1, 1, 1, 1) = 0, (1, 1, 1, 2) = 0, (1, 1, 1, 3) = 0, (1, 1, 1, 4) = 0, (1, 1, 2, 1) \\ = 0, (1, 1, 2, 2) = 0, (1, 1, 2, 3) = 0, (1, 1, 2, 4) = 0, (1, 1, 3, 1) = 0, (1, 1, 3, 2) = 0, (1, 1, 3, 3) = 0, (1, 1, 3, 4) = 0, (1, 1, 4, 1) = 0, (1, 1, 4, 2) = 0, (1, 1, 4, 3) = 0, (1, 1, 4, 4) = 0, (1, 2, 1, 1) = 0, (1, 2, 1, 2) = 0, (1, 2, 1, 3) = 0, (1, 2, 1, 4) = 0, (1, 2, 2, 1) = 0, (1, 2, 2, 2) = 0, (1, 2, 2, 3) = 0, (1, 2, 2, 4) = 0, (1, 2, 3, 1) = 0, (1, 2, 3, 2) = 0, (1, 2, 3, 3) = 0, (1, 2, 3, 4) = 0, (1, 2, 4, 1) = 0, (1, 2, 4, 2) = 0, (1, 2, 4, 3) = 0, (1, 2, 4, 4) = 0, (1, 3, 1, 1) = 0, (1, 3, 1, 2) = 0, (1, 3, 1, 3) = 0, (1, 3, 2, 1) = 0, (1, 3, 2, 2) = 0, (1, 3, 2, 3) = 0, (1, 3, 2, 4) = 0, (1, 3, 3, 1) = 0, (1, 3, 3, 2) = 0, (1, 3, 3, 3) = 0, (1, 3, 3, 4) = 0, (1, 3, 4, 1) = 0, (1, 3, 4, 2) = 0, (1, 3, 4, 3) = 0, (1, 3, 4, 4) = 0, (1, 4, 1, 1) = 0, (1, 4, 1, 2) = 0, (1, 4, 1, 3) = 0, (1, 4, 1, 4) = 0, (1, 4, 2, 1) = 0, (1, 4, 2, 2) = 0, (1, 4, 2, 3) = 0, (1, 4, 2, 4) = 0, (1, 4, 3, 1) = 0, (1, 4, 3, 2) = 0, (1, 4, 3, 3) = 0, (1, 4, 3, 4) = 0, (1, 4, 4, 1) = 0\end{array}\right]\right)\right)$ (16)

$$\dots, (1, 1, 2, 1) = 0, (1, 1, 2, 2) = 0, (1, 1, 2, 3) = 0, (1, 1, 2, 4) = 0, (1, 1, 3, 1) = 0, (1, 1, 3, 2) = 0, (1, 1, 3, 3) = 0, (1, 1, 3, 4) = 0, (1, 1, 4, 1) = 0, (1, 1, 4, 2) = 0, (1, 1, 4, 3) = 0, (1, 1, 4, 4) = 0,$$

$$(1, 2, 1, 1) = 0, (1, 2, 1, 2) = 0, (1, 2, 1, 3) = 0, (1, 2, 1, 4) = 0, (1, 2, 2, 1) = 0, (1, 2, 2, 2) = 0, (1, 2, 2, 3) = 0, (1, 2, 2, 4) = 0, (1, 2, 3, 1) = 0, (1, 2, 3, 2) = 0, (1, 2, 3, 3) = 0, (1, 2, 3, 4) = 0, (1, 2, 4, 1) = 0, (1, 2, 4, 2) = 0, (1, 2, 4, 3) = 0, (1, 2, 4, 4) = 0, (1, 3, 1, 1) = 0,$$

$$(1, 3, 1, 2) = 0, (1, 3, 1, 3) =$$

$$= \frac{1}{1 + u \text{Heaviside}(u)} (-2 \text{Dirac}(u) - u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u)), (1, 3, 1, 4) = 0, (1, 3, 2, 1) = 0, (1, 3, 2, 2) = 0, (1, 3, 2,$$

$$3) = 0, (1, 3, 2, 4) = 0, (1, 3, 3, 1) =$$

$$- \frac{1}{1 + u \text{Heaviside}(u)} (-2 \text{Dirac}(u) - u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u)), (1, 3, 3, 2) = 0, (1, 3, 3, 3) = 0, (1, 3, 3, 4) = 0, (1, 3, 4,$$

$$1) = 0, (1, 3, 4, 2) = 0, (1, 3, 4, 3) = 0, (1, 3, 4, 4) = 0, (1, 4, 1, 1) = 0, (1, 4, 1, 2) = 0, (1,$$

$$4, 1, 3) = 0, (1, 4, 1, 4) =$$

$$- \frac{1}{-1 + u \text{Heaviside}(u)} (2 \text{Dirac}(u) + u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u)), (1, 4, 2, 1) = 0, (1, 4, 2, 2) = 0, (1, 4, 2, 3) = 0, (1, 4, 2,$$

$$4) = 0, (1, 4, 3, 1) = 0, (1, 4, 3, 2) = 0, (1, 4, 3, 3) = 0, (1, 4, 3, 4) = 0, (1, 4, 4, 1) = 0$$

$$\begin{aligned}
&= \frac{1}{-1 + u \operatorname{Heaviside}(u)} (2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
&\quad + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u)), (1, 4, 4, 2) = 0, (1, 4, 4, 3) = 0, (1, 4, 4, 4) = 0, (2, 1, 1, \\
&\quad 1) = 0, (2, 1, 1, 2) = 0, (2, 1, 1, 3) = 0, (2, 1, 1, 4) = 0, (2, 1, 2, 1) = 0, (2, 1, 2, 2) = 0, (2, \\
&\quad 1, 2, 3) = 0, (2, 1, 2, 4) = 0, (2, 1, 3, 1) = 0, (2, 1, 3, 2) = 0, (2, 1, 3, 3) = 0, (2, 1, 3, 4) \\
&\quad = 0, (2, 1, 4, 1) = 0, (2, 1, 4, 2) = 0, (2, 1, 4, 3) = 0, (2, 1, 4, 4) = 0, (2, 2, 1, 1) = 0, (2, 2, \\
&\quad 1, 2) = 0, (2, 2, 1, 3) = 0, (2, 2, 1, 4) = 0, (2, 2, 2, 1) = 0, (2, 2, 2, 2) = 0, (2, 2, 2, 3) = 0, \\
&\quad (2, 2, 2, 4) = 0, (2, 2, 3, 1) = 0, (2, 2, 3, 2) = 0, (2, 2, 3, 3) = 0, (2, 2, 3, 4) = 0, (2, 2, 4, 1) \\
&\quad = 0, (2, 2, 4, 2) = 0, (2, 2, 4, 3) = 0, (2, 2, 4, 4) = 0, (2, 3, 1, 1) = 0, (2, 3, 1, 2) = 0, (2, 3, \\
&\quad 1, 3) = 0, (2, 3, 1, 4) = 0, (2, 3, 2, 1) = 0, (2, 3, 2, 2) = 0, (2, 3, 2, 3) = 0, (2, 3, 2, 4) = 0, \\
&\quad (2, 3, 3, 1) = 0, (2, 3, 3, 2) = 0, (2, 3, 3, 3) = 0, (2, 3, 3, 4) = 0, (2, 3, 4, 1) = 0, (2, 3, 4, 2) \\
&\quad = 0, (2, 3, 4, 3) = 0, (2, 3, 4, 4) = 0, (2, 4, 1, 1) = 0, (2, 4, 1, 2) = 0, (2, 4, 1, 3) = 0, (2, 4, \\
&\quad 1, 4) = 0, (2, 4, 2, 1) = 0, (2, 4, 2, 2) = 0, (2, 4, 2, 3) = 0, (2, 4, 2, 4) = 0, (2, 4, 3, 1) = 0, \\
&\quad (2, 4, 3, 2) = 0, (2, 4, 3, 3) = 0, (2, 4, 3, 4) = 0, (2, 4, 4, 1) = 0, (2, 4, 4, 2) = 0, (2, 4, 4, 3) \\
&\quad = 0, (2, 4, 4, 4) = 0, (3, 1, 1, 1) = 0, (3, 1, 1, 2) = 0, (3, 1, 1, 3) = \\
&\quad - \frac{1}{1 + u \operatorname{Heaviside}(u)} (-2 \operatorname{Dirac}(u) - u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
&\quad + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u)), (3, 1, 1, 4) = 0, (3, 1, 2, 1) = 0, (3, 1, 2, 2) = 0, (3, 1, 2, \\
&\quad 3) = 0, (3, 1, 2, 4) = 0, (3, 1, 3, 1) \\
&= \frac{1}{1 + u \operatorname{Heaviside}(u)} (-2 \operatorname{Dirac}(u) - u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
&\quad + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u)), (3, 1, 3, 2) = 0, (3, 1, 3, 3) = 0, (3, 1, 3, 4) = 0, (3, 1, 4, \\
&\quad 1) = 0, (3, 1, 4, 2) = 0, (3, 1, 4, 3) = 0, (3, 1, 4, 4) = 0, (3, 2, 1, 1) = 0, (3, 2, 1, 2) = 0, (3,
\end{aligned}$$

$$\begin{aligned}
& (2, 1, 3) = 0, (3, 2, 1, 4) = 0, (3, 2, 2, 1) = 0, (3, 2, 2, 2) = 0, (3, 2, 2, 3) = 0, (3, 2, 2, 4) \\
& = 0, (3, 2, 3, 1) = 0, (3, 2, 3, 2) = 0, (3, 2, 3, 3) = 0, (3, 2, 3, 4) = 0, (3, 2, 4, 1) = 0, (3, 2, \\
& 4, 2) = 0, (3, 2, 4, 3) = 0, (3, 2, 4, 4) = 0, (3, 3, 1, 1) = 0, (3, 3, 1, 2) = 0, (3, 3, 1, 3) = 0, \\
& (3, 3, 1, 4) = 0, (3, 3, 2, 1) = 0, (3, 3, 2, 2) = 0, (3, 3, 2, 3) = 0, (3, 3, 2, 4) = 0, (3, 3, 3, 1) \\
& = 0, (3, 3, 3, 2) = 0, (3, 3, 3, 3) = 0, (3, 3, 3, 4) = 0, (3, 3, 4, 1) = 0, (3, 3, 4, 2) = 0, (3, 3, \\
& 4, 3) = 0, (3, 3, 4, 4) = 0, (3, 4, 1, 1) = 0, (3, 4, 1, 2) = 0, (3, 4, 1, 3) = 0, (3, 4, 1, 4) = 0, \\
& (3, 4, 2, 1) = 0, (3, 4, 2, 2) = 0, (3, 4, 2, 3) = 0, (3, 4, 2, 4) = 0, (3, 4, 3, 1) = 0, (3, 4, 3, 2) \\
& = 0, (3, 4, 3, 3) = 0, (3, 4, 3, 4) = 0, (3, 4, 4, 1) = 0, (3, 4, 4, 2) = 0, (3, 4, 4, 3) = 0, (3, 4, \\
& 4, 4) = 0, (4, 1, 1, 1) = 0, (4, 1, 1, 2) = 0, (4, 1, 1, 3) = 0, (4, 1, 1, 4) = 0, \\
& = \frac{1}{-1 + u \operatorname{Heaviside}(u)} (2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
& + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u)), (4, 1, 2, 1) = 0, (4, 1, 2, 2) = 0, (4, 1, 2, 3) = 0, (4, 1, 2, \\
& 4) = 0, (4, 1, 3, 1) = 0, (4, 1, 3, 2) = 0, (4, 1, 3, 3) = 0, (4, 1, 3, 4) = 0, (4, 1, 4, 1) = 0, \\
& - \frac{1}{-1 + u \operatorname{Heaviside}(u)} (2 \operatorname{Dirac}(u) + u \operatorname{Dirac}(1, u) + 2 \operatorname{Heaviside}(u) u \operatorname{Dirac}(u) \\
& + u^2 \operatorname{Heaviside}(u) \operatorname{Dirac}(1, u)), (4, 1, 4, 2) = 0, (4, 1, 4, 3) = 0, (4, 1, 4, 4) = 0, (4, 2, 1, \\
& 1) = 0, (4, 2, 1, 2) = 0, (4, 2, 1, 3) = 0, (4, 2, 1, 4) = 0, (4, 2, 2, 1) = 0, (4, 2, 2, 2) = 0, (4, \\
& 2, 2, 3) = 0, (4, 2, 2, 4) = 0, (4, 2, 3, 1) = 0, (4, 2, 3, 2) = 0, (4, 2, 3, 3) = 0, (4, 2, 3, 4) \\
& = 0, (4, 2, 4, 1) = 0, (4, 2, 4, 2) = 0, (4, 2, 4, 3) = 0, (4, 2, 4, 4) = 0, (4, 3, 1, 1) = 0, (4, 3, \\
& 1, 2) = 0, (4, 3, 1, 3) = 0, (4, 3, 1, 4) = 0, (4, 3, 2, 1) = 0, (4, 3, 2, 2) = 0, (4, 3, 2, 3) = 0, \\
& (4, 3, 2, 4) = 0, (4, 3, 3, 1) = 0, (4, 3, 3, 2) = 0, (4, 3, 3, 3) = 0, (4, 3, 3, 4) = 0, (4, 3, 4, 1) \\
& = 0, (4, 3, 4, 2) = 0, (4, 3, 4, 3) = 0, (4, 3, 4, 4) = 0, (4, 4, 1, 1) = 0, (4, 4, 1, 2) = 0, (4, 4, \\
& 1, 3) = 0, (4, 4, 1, 4) = 0, (4, 4, 2, 1) = 0, (4, 4, 2, 2) = 0, (4, 4, 2, 3) = 0, (4, 4, 2, 4) = 0, \\
& (4, 4, 3, 1) = 0, (4, 4, 3, 2) = 0, (4, 4, 3, 3) = 0, (4, 4, 3, 4) = 0, (4, 4, 4, 1) = 0, (4, 4, 4, 2) \\
& = 0, (4, 4, 4, 3) = 0, (4, 4, 4, 4) = 0]]]$$

displayGR(`weyl` , Weyl);

```
> displayGR(`Weyl`, WEYL);
```

The Weyl Tensor

C1313

$$= \frac{1}{1 + u \text{Heaviside}(u)} (-2 \text{Dirac}(u) - u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u))$$

C1414 =

$$- \frac{2 \text{Dirac}(u) + u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u)}{-1 + u \text{Heaviside}(u)}$$

character : [-1, -1, -1, -1] (17)

$$1, 3, 1, 3 \quad (18)$$

> C1313:=(-2*Dirac(u)-u*Dirac(1, u)+2*Heaviside(u)*u*Dirac(u)+u^2*Heaviside(u)*Dirac(1, u))/(1+u*Heaviside(u));

C1313 :=

$$\frac{-2 \text{Dirac}(u) - u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u)}{1 + u \text{Heaviside}(u)} \quad (19)$$

> C1414:=-(2*Dirac(u)+u*Dirac(1, u)+2*Heaviside(u)*u*Dirac(u)+u^2*Heaviside(u)*Dirac(1, u))/(-1+u*Heaviside(u));

C1414 :=

$$- \frac{2 \text{Dirac}(u) + u \text{Dirac}(1, u) + 2 \text{Heaviside}(u) u \text{Dirac}(u) + u^2 \text{Heaviside}(u) \text{Dirac}(1, u)}{-1 + u \text{Heaviside}(u)} \quad (20)$$

> psi:=simplify(expand(-C1313+C1414)/2); #Psi_4

$$\psi := \text{Dirac}(u) \quad (21)$$