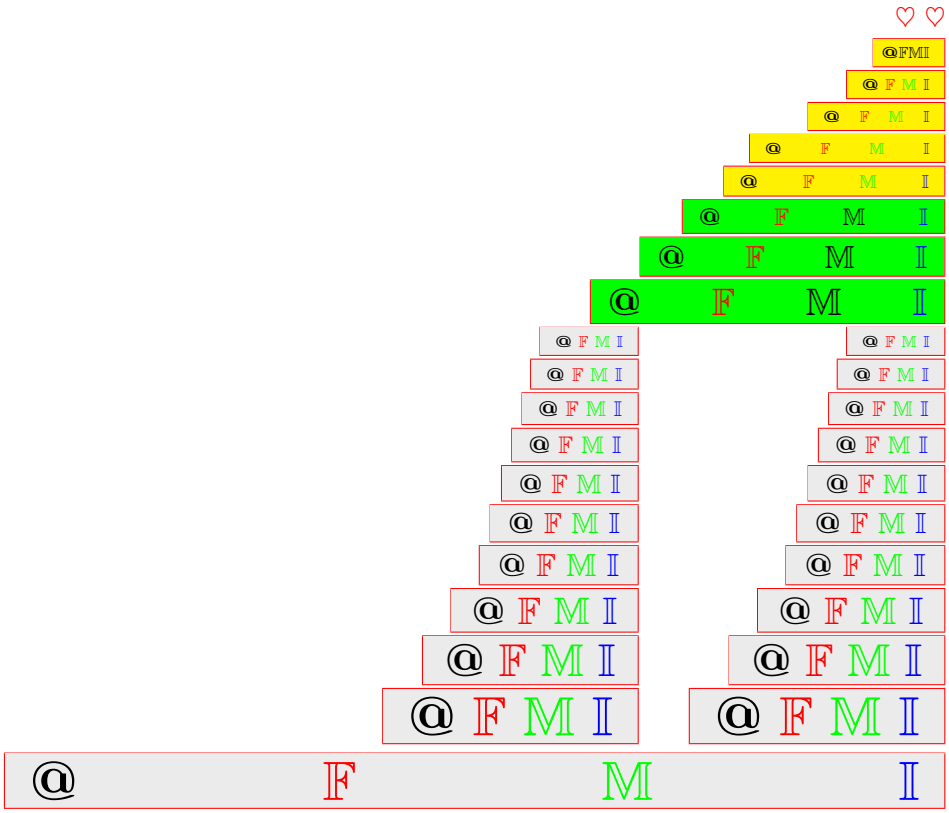
Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 2, No. 1, (July 2011), pp. 81- 89
ISSN 2093–9310
http://www.afmi.or.kr

Fuzzy instead of discrete

P. Kambaki-Vougioukli, A. Karakos, N. Lygeros, T. Vougiouklis



Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 2, No. 1, July 2011

Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 2, No. 1, (July 2011), pp. 81- 89
ISSN 2093–9310
http://www.afmi.or.kr

Fuzzy instead of discrete

P. Kambaki-Vougioukli, A. Karakos, N. Lygeros, T. Vougiouklis

Received 5 January 2011; Revised 21 January 2011; Accepted 23 January 2011

Abstract. Vougiouklis and Vougiouklis proposed the replacement of the usually used in questionnaires Likert scale with a bar. In fact, with this replacement a discrete situation is replaced by a fuzzy one. In present paper there are discussed certain advantages of the bar compared and contrasted with the scale during both the filling-in and processing stages of a questionnaire. Finally, the suggested method is applied in a *Course and Teaching Evaluation* process by the students of Democritus University of Thrace.

2010 AMS Classification: 03E72, 08A72

Keywords: Fuzzy sets, Likert scale, bar, questionnaires.

Corresponding Author: T. Vougiouklis (tvougiou@eled.duth.gr)

1. Research background

Mathematical models are often used in almost every field of empirical research in order to reinforce the reliability of each research. Moreover, mathematicalisation, so to say, of the problems could make it possible to recognize and compare the results, see [10]. This is because representing an actual research object or a phenomenon in numbers is widely accepted to be the simplest and the most recognizable way of reading the actual results.

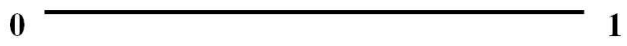
As far as any empirical research is concerned, three are the main stages to be identified, namely: (a) design, (b) implementation and (c) processing of the results. As for the main tools involved, *the questionnaire* is undoubtedly of crucial importance and its compilation demands special concern and careful consideration, as it might affect the reliability of the final results of the research. The type of scale normally used is the Likert scale and it is widely employed in questionnaires in order to measure respondents' -subjects' attitudes by asking the extent to which they agree or disagree with a particular question or statement. A typical scale might be *"absolutely agree, almost agree, not sure / undecided, rather disagree, completely disagree"*.

Likert scales may appear to be easy to analyze, nevertheless we claim that there are important issues that it is worth being considered by a data analyst. Grading a variant depends both on its actual nature as well as on the researcher's judgment [1, 4, 5, 6, 7]. More specifically, there are certain shortcomings identified in this type of scales, namely: (a) The range of the scale each time, in other words how many different subdivisions, or grades, will be used each time, is upon the researcher to decide. Such a decision is not at all an easy and straightforward one to make since it is completely different to have 3, 4 or 5 steps in a scale. The choice of any subdivision may have certain shortcomings, for example lack of a medium choice in a 4-scale. (b) Another shortcoming of a typical Likert scale concerns the difficulty on the part of the researcher to explain to the subjects the fine differences between the different subdivisions, as many researchers report that it takes them more time to explain the subjects what each subdivision on their scale represents rather than accomplish the actual test - a problem not purely linguistic of course but rather social and psychological. (c) The existence of only one possibility of working the results out, the one the researcher decided to establish when designing the experiment, might prove to be frustrating, because it might deprive the researcher of the possibility to explore other parameters which might crop up in the process or, try different subdivisions for a more accurate calculation. Clearly, it is always the kind of research that will decide the type of scale to be used, however if the researcher wants to investigate some other parameters s/he had not provided for, or if s/he wants to view some aspects from a different point of view, s/he might want to re-apply the test with a different scale. In this case, though, s/he will have to begin the process of familiarization of her/his subjects with the new categorization. Consequently, in every case, the problem is the establishment and evaluation of the 'unit'.

1.1. **'Bar' instead of scale.** In order to minimize this type of shortcomings, we propose the substitution of the Likert scale with a bar, as suggested by Vougiouklis and Vougiouklis in [6]. We strongly believe that this substitution will make things simpler and easier for both the subjects of an empirical research and the researcher in both the stage of design and the processing of the results as it is really flexible. Moreover, the application of the bar opens a window towards the use of fuzzy sets in the process of empirical research.

More specifically, we adopt and propose the following:

"In every question, substitute the Likert scale with 'the bar' whose poles are defined with '0' on the left and '1' on the right:



The subjects/participants are asked, instead of deciding and checking a specific grade on the scale, to cut the bar at any point they feel best expresses their answer to the specific question".

1.2. **Certain advantages of the use of the 'bar' in linguistic research.** The important subject of the linguistic parameter of the scales and its correlation to the fuzzy theory has been widely investigated by Zadeh [11, 12], the founder of the fuzzy

theory. Zadeh has characterized the related variable "linguistic variable" accepting in this way the important role of linguistic theory in the development of fuzzy sets, something extremely important for linguistics. Yet, the linguistic escalation of the question parameter is not an easy job to accomplish (see Kambaki-Vougioukli [5]), as it depends upon factors other than purely linguistic ones, including all psychological, educational, economic and social status, age, gender and/or even language of the subjects. Consequently, we feel that, in the normally used questionnaires with Likert scales, it is rather risky to claim that the grades of the scale are clearly pinpointed and comprehended by all subjects. For example, in a scale where 3='absolutely sure', 2='almost sure', 1='rather unsure' and 0='completely unsure', we cannot say that everybody distinguishes between 3 and 2 or 0 and 1 in the same way. Even more so, some subjects would like something in the middle or, even, something between 1 and 2 or 2 and 3 etc, i.e. a more detailed escalation. It is as if you give people a flight of stairs the height of which is good only for people who are, say, 180cm tall while the shorter or younger ones will have trouble using them.

By contrast, we have every reason top believe that there is no need to demand from our subjects to try to distinguish the difference between the grades of a scale, an attempt which is hard anyway and might very well be in vain. Furthermore, we are sure that no special training will be necessary for our subjects so as to be able to understand how to cut the bar proposed as they can cut it intuitively, without much verbal explanation on the part of the researcher. It is as if having somebody on a wheelchair going up an inclined plane using a ramp rather than a flight of stairs. Consequently, the psychological factor- the most important one - could be easily controlled as the subject is asked to cut the bar 01 based only on intuition, which actually defines the most accurate point that specific moment the decision is made. The bar, in other words, gives access to a fuzzy attitude since it requires a mapping in the space 01 instead of a discreet answer 0 or 1.

Another issue that is worth referring to is the following: if a subject is used to filling in questionnaires using the Likert scales, h/se is expected to have created in her/his mind a scale of 3, 4, or 5 grades, or, even, of 10 or 100 grades or steps by means of which s/he evaluates the questions posed. Normally 10 is used for grading or marking while 100 for percentages. Obviously, the per cent answer is closer to that of the bar, since the 100-grade escalation of the short bar is closer to that of a continuum than the 10-grade one.

As Lygeros [7] points out "the Vougiouklis and Vougiouklis innovation concerning the substitution of the scale with the bar is originated from the contention between continuum and discreet. The tool proposed is neither another one among the others suggested in the methodology of questionnaires nor some excuse in order that a theory should be artificially used and finally come up with the same results. By contrast, the Vougiouklis and Vougiouklis bar is trying to overcome, in a skilful mathematical way, methodological shortcomings of simplification created by a model such as the Likert scale. Every scale, using a discreet space for data analysis, creates by definition problems of orientation while trying to function in a space of finite dimensions. The specific choices offered by Likert scales usually concern 5 points in an absolute hierarchy, which appears to be some kind of simplification in statistics, yet it only consists a type of thought decay. If such a statement sounds rather

provocative, consider the fact that thought should be limited within the elements of a finite set. Simplification results in naivety. The problem is oversimplification".

1.3. **Data processing.** Questionnaire procession using the bar gives the initiative to the researcher, who has the chance to 'escalate' the answers without having to decide in advance how many grades there will be finally needed so as to be able to identify the parameters and clarify the differences between the grades. Even more so, there is a good degree of flexibility in establishing balanced or imbalanced scales taking into consideration the needs of the specific research each time. More specifically, after the researcher has collected all the filled-in questionnaires, s/he will be able to process and access them in numerous ways without having to repeat the test putting the subjects in a new time consuming process and risking the reliability of results. In addition, another shortcoming of a repetition of a test, that of the subjects' knowing/guessing what is expected from them, is overcome. Using the bar minimizes such risks and, more important, reinforces the objectivity of it as it gives more space to mathematical processing of the results.

As it is widely accepted, one of the main characteristics of applied mathematics is the ability of different approaches, the ability of simplification of the form. This is the reason why linear models are preferred and there is a tendency of changing continuum into discreet and vice-versa. Clearly, the bar offers the possibility of accurate processing which is the optimum for the researcher: from discrete into continuous and, even more, from single valued into multivalued or fuzzy [2, 3, 8, 9, 10]. This is pointed out by Lygeros [7], who makes it clear that: "Our approach, however, does not focus on this specific aspect but on the mathematical possibilities involved in Vougiouklis and Vougiouklis proposal concerning the data analysis rather than the actual choices-answers of the subjects. Vougiouklis and Vougiouklis bar secures the use of Differential Calculus. In this simple, yet fundamental, way the bar extinguishes the problems of internal limits and allows us to handle things as alternatives on the real line, provided there isomorphy with the space]0,1[. Such a fact implies that the same results of our subjects; choices could be studied via a variety of tools. Moreover, they offer another possibility, that of the multiple use of statistical tools which reinforce the resilience of the research results. In other words, the bar could also function as a tool to control the quality of other scales as it is the case with Weschler (SD15), Stanford-Binet (SD16) $\kappa\alpha\iota$ Cattell (SD24). Finally, as Vougiouklis and Vougiouklis point out, the discreet-continuum change could be possibly followed by the single value to multi value change with possibilities of realistic simulation of human choices."

The use of the multivalued maps and the hyperstructure theory, which is closely related with fuzzy set theory [2, 3, 8, 9], could be of assistance to a lot of applications including data processing and language teaching (see application in [10]. The issue of the people's attitudes towards the bar as compared to that of Likert scales could be also checked by asking participants who are used filling in questionnaires using Likert scales to specify whether the bar or the other type of scale makes things easier for them. In this way we could have an evaluation of our proposal.

2. Purpose and rationale

Bearing in mind the lack of previous research concerning alternative methods of data processing in questionnaires rather than scales, the purpose of the present study was to determine whether the use of the bar instead of a scale, more specifically Likert scale, could be more effective and easily plausible. Our objective was also to test our subjects' reaction and their attitude towards the method suggested.

Within the frame of evaluation of the Department of Primary Education of Democritus University of Thrace, 1st year students were allocated questionnaires of subject evaluation concerning Geometry and Algebra where (a) the scale was substituted by the bar and (b) they were asked to specify whether they preferred the bar or the scale.

3. Method

3.1. **Participants.** There were two groups of students: Group 1 consisted of 143 students, who had just finished the first term of their first year of studies while Group 2 consisted of 109 (different) students, who had just accomplished the second term of their first year of studies.

3.2. **Instrumentation.** For the successful elicitation of the data the use of self report questionnaires was considered the most appropriate. The questionnaires consisted of 65 questions in which instead of the normally used Likert scale, we used the bar [01].

Finally, there was an extra question at the end of each questionnaire, asking the participants whether they preferred the bar or the Likert scale.

3.3. **Procedure.** ***Group 1*** : By the end of the first term, all the 143 students filled in the questionnaire consisting of 65 items concerning the taught subject of Algebra. In all questions there was used a bar instead of a 6-grade Likert scale used in any other subject evaluation. Questions included the students' judgment of the instructors' effectiveness, the quality of the subject, the evaluation of the coursebooks, laboratories etc. Finally, they had to state whether they preferred the bar to a usual Likert scale.

Group 2 : Similarly, by the end of the second term, 109 students filled in the questionnaire consisting of 65 items concerning the taught subject of Geometry. In all questions there was used a bar instead of a 6-grade Likert scale used in any other subject evaluation. Questions included the students' judgment of the instructors' effectiveness, the quality of the subject, the evaluation of the coursebooks, laboratories etc. Finally, they had to state whether they preferred the bar to a usual Likert scale.

3.4. **Data analysis.** In order to study the difference amongst various possible ways of codification of results, we chose the following three:

A. codification of the answers into 4 equal-length spaces (4 classes)

B. codification of the answers into 5 equal-length spaces (5 classes)

C. codification of the answers into 5 equal-area spaces according to Gauss distribution, that is to say 5 classes with limit value 0.39, 0.47, 0.53, 0.61 and 1.

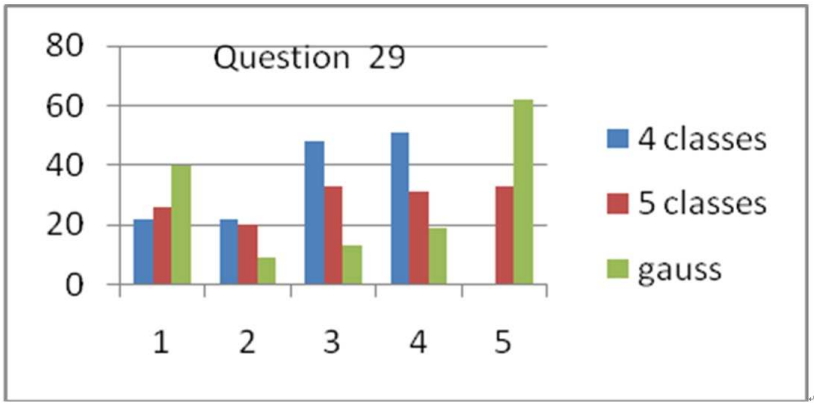


FIGURE 1. Bar-graph 1

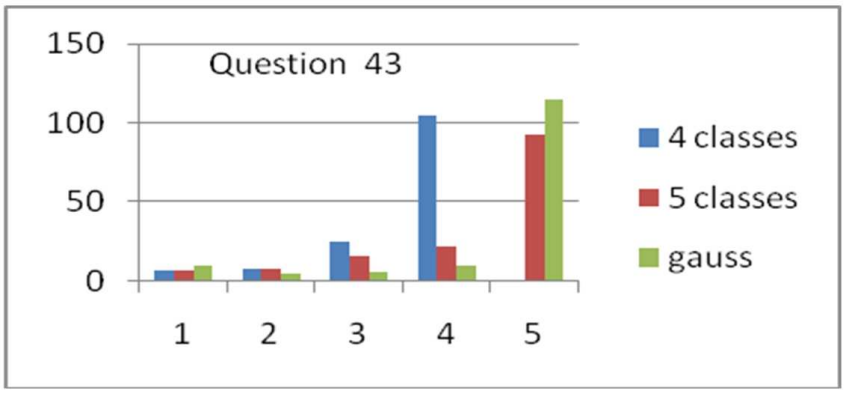


FIGURE 2. Bar-graph 2

4. RESULTS-DISCUSSION

Indicatively, in Bar-graphs 1 and 2 (see Figures 1 and 2), there are presented the processing results of two of the questions, namely questions 29 *"Specify the degree of interest aroused by the teacher him/herself"*, and 43 *"Specify the degree of teacher's correspondence to his/her teaching duties"*, from the Algebra questionnaire, where the following is observed : when the answers are divided or codified into equal-length spaces, no substantial difference is observed in classes 1 and 2 while in any other class the difference is obvious. This might be due to the fact that answers in high classes do not substantially differ amongst themselves concerning the limits defined and this fact has as a result these answers to shift from one class to the other depending on the limits defined each time.

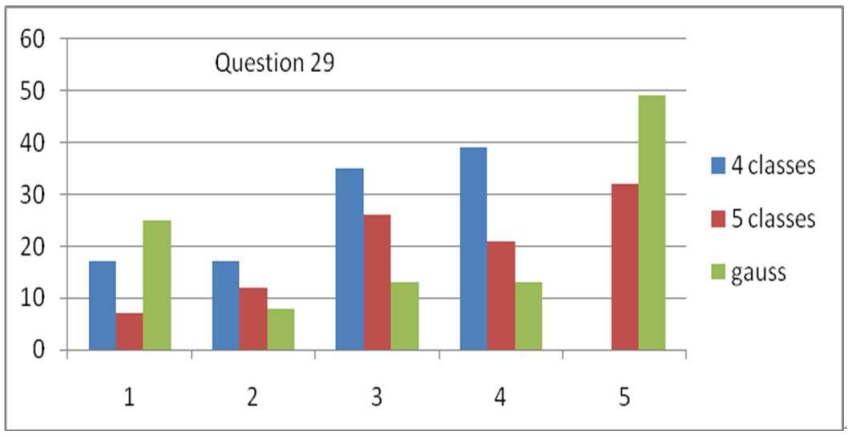


FIGURE 3. Bar-graph 3

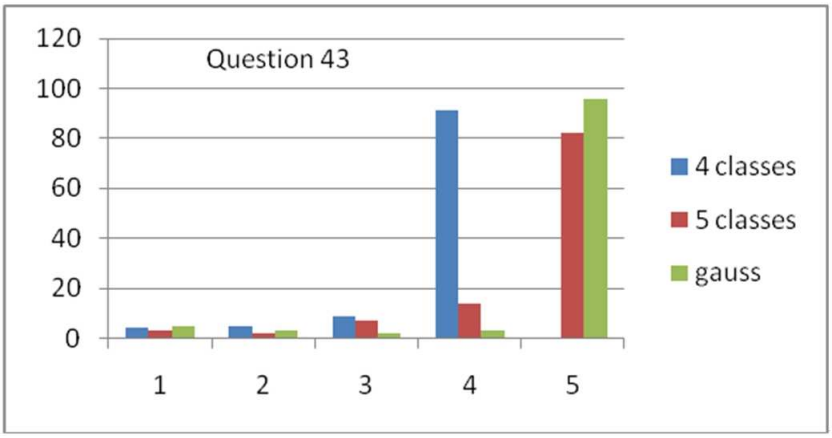


FIGURE 4. Bar-graph 4

Similarly, when we choose the Gauss distribution, we can easily see the different distribution of the answers on the 5 classes, because all the answers seem to be crowded on the two extreme classes.

In bar-graphs 3 and 4 (see Figures 3 and 4) we have the processed results of questions 29 and 43 of the Geometry questionnaire, where, once more, we can observe that when the answers are divided or codified into equal-length spaces, no substantial difference is observed while in every other class the difference is substantial and obvious.

As far as the extra question about students' attitude towards the use of the bar as compared, or contrasted, to Likert scale, there was a massive preference of the bar :

236 out of 252 answered the specific question as follows: 197, or 83,47 %, prefer the bar and 39, or 16,53 % prefer Likert scale.

5. Conclusion

From the above discussion, the conclusion could be easily drawn that the use of the bar instead of a scale gives the researcher the chance to handle the questionnaire in a more flexible and dynamic way, taking into account the special difficulties and characteristics of each different research as well as the need of greater or smaller numbers of classes of the variables (questions). In every case, results derived by the use of a bar could be easily adapted to results derived by using a scale, making them comparable, while the opposite is impossible. This is because any answer given on a bar can be transferred on any scale, while answers given on a specific scale not only cannot be transferred on a bar but the opposite is impossible. Consequently, if the researcher wishes to compare results using different codifications in order to find out whether different conclusions could be derived, s/he has no other choice but repeat the same test with all the above mentioned shortcomings of a repetition of the same experiment. To conclude, we hold that the suggested bar may offer solutions never thought of before and help researchers consider result processing from a different viewpoint.

References

[1] L. Cohen and L. Manion, Research Methods in Education, Routledge, 1994.
[2] B. Davvaz and V. Leoreanu, Hyperring Theory and Applications, Int. Academic Press, 2007.
[3] P. Corsini and V. Leoreanu, Applications of Hypergroup Theory, Kluwer Academic Publishers, 2003.
[4] P. Kambaki-Vougioukli, Greek and english readers' accuracy and confidence when inferencing meanings of unknown words. Proc. 6th Inter. Symposium on the Description and/or Comparison of English and Greek (1992) 89–112.
[5] P. Kambaki-Vougioukli, Language and Mathematical Models, Kyriakidis, 2009 (in Greek)
[6] P. Kambaki-Vougioukli and T. Vougiouklis, Bar instead of scale, Ratio Sociologica 3 (2008) 49–56.
[7] N. Lygeros, Interrogations fondamentales sur la méthodologie du questionnaire, Perfection 1055/2009.
[8] T. Vougiouklis, Hyperstructures and their Representations, Monographs in Math., Hadronic, 1994.
[9] T. Vougiouklis, ∂-operations and Hv-fields, Acta Math. Sin., 24(7) (2008) 1067–1078.
[10] T. Vougiouklis and P. Kambaki, Algebraic models in applied research, Jordan J. Math. Statistics 1(1) (2008) 78–87.
[11] L. A. Zadeh, Fuzzy sets, Inform. Control, 12(2) (1965) 94–102.
[12] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Inform. Sci. 8 (1975) 199–249.

P. Kambaki-Vougioukli (pekavou@helit.duth.gr) – Democritus University of Thrace, Greece

A. Karakos (karakos@ee.duth.gr) – Democritus University of Thrace, Greece

N. Lygeros (nlygeros@gmail.com) – 14 Avenue Condoriet, 69100 Villeurbanne, France

T. VOUGIOUKLIS (tvougiou@eled.duth.gr) – School of Education, Democritus University of Thrace, 68100 Nea Hili, Alexandroupolis, Greece

Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 2, No. 1, (July 2011), pp. 1- 130
ISSN 2093–9310
http://www.afmi.or.kr

Contents