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$$p \in \Pi \wedge 2p+1 \in \Pi \Rightarrow p \in SG$$

$SG := \{2,3,5,11,23,29,41,53,83,89,113,131,...\}$

Conjecture : $\#SG(\leq N) \sim 2C_2 \int_2^N \frac{dx}{\ln x \ln 2x+1} \sim \frac{2C_2 N}{(\ln N)^2}$

$C_2 = 0.660168158$

N	Computations		Estimations
1.000	37	<	39
100.000	1171	>	1166
10.000.000	56032	<	56128
100.000.000	423140	<	423295
1.000.000.000	3308859	>	3307888
10.000.000.000	26569515	>	26568824

Theorem (Euleur-Lagrange)

$p \equiv 3 \text{ mod } 4 \in \Pi$

$2p+1 \in \Pi \iff 2p+1 \mid Mp$

Proof : $(\Rightarrow)$

$q = 2p+1 \in \Pi$

$p \equiv 3 \bmod 4 \Rightarrow q \equiv 7 \bmod 8$

$n^2 \equiv 2 \bmod q$

$2^p \equiv 2^{q-1/2} \equiv n^{q-1} \equiv 1 \bmod q$

$q \mid Mp$

$\Leftarrow$ By contradiction

$2p+1 \mid Mp$ *If* $2p+1 \notin \Pi$ *then* $q \in \Pi \wedge q \mid 2p+1$

q its least prime $\Rightarrow 2^p \equiv 1 \bmod q$

$p \mid q-1 \Rightarrow q > p \Rightarrow (2p+1)+1 > q^2 > p^2$

As $p > 2$, we get a contradiction.