Relations between Germain and Ramanujan Numbers

N. Lygeros, O. Rozier

$$p \in \Pi \wedge 2p+1 \in \Pi \Rightarrow p \in SG$$

$SG := \{2,3,5,11,23,29,41,53,83,89,113,131,...\}$

$P^{+} := \{p > 2 : p \in \Pi / \tau(p) \equiv 0[\mathrm{p}+1]\}$

$P^{-} := \{p > 2 : p \in \Pi / \tau(p) \equiv 2[\mathrm{p}-1]\}$

$A := 2^{14}.3^{7}.5^{3}.7^{2}.23.691$

$B := 2^{11}.3^{6}.5^{3}.7.691$

$P_0^{+} := \{p > 2 : p \in \Pi / p+1 \mid A\}$

$P_0^{-} := \{p > 2 : p \in \Pi / p-1 \mid B\}$

Lemma : $P_0^{+} \subset P^{+}$ Lemma : $P_0^{-} \subset P^{-}$

$$P_0^{+} = \left\{3,5,7,11,13,17,19,23,29,31,41,47,53,59,71,79,83,89,97,...,\frac{1}{6}A-1,\frac{1}{5}A-1\right\}$$

$$P_0^{-} = \left\{3,5,7,11,13,17,19,29,31,37,41,43,61,71,73,97,...,\frac{1}{32}B+1,B+1\right\}$$

$SG_0 = \{p > 2 : p \in SG\}$

$\otimes$ $\{3,5,11\} \subseteq P_0^{-}$ $\otimes 23 \notin P_0^{-} : 23-1=22=2.11,\ 11 \nmid \mathrm{B},\ 23 \notin P_0^{-}$

$\otimes$ $83 \in SG_0$ $83 \notin P_0^{-}$

As 83-1=82=2.41, $41 \nmid B$, $83 \notin P_0^{-}$

$\otimes$ $89 \in SG_0$ $89 \notin P_0^{-1}$

As $89-1=88=2^{3}.11$, $11 \nmid B$, $89 \notin P_0^{-1}$

$\otimes\{3,5,11,23,29,41,53,83,89\} \subseteq P_0^{+}$

$\otimes 113 \in SG_0$ $113 \notin P_0^{+}$

As 113+1=114=23.19, $19 \nmid \mathrm{A}$, $113 \notin P_0^{+}$

$\otimes 131 \in SG_0$ $131 \notin P_0^{+}$

As 131+1=132= $2^{2}.3.11$, $11 \nmid \mathrm{A}$, $131 \notin \mathrm{P}_0^{+}$

As $\tau(113) = -85146862638$, $114 \nmid \tau(113) \Rightarrow 113 \notin P^{+}$

As $\tau(131) = 631528759932$, $132 \nmid \tau(131) \Rightarrow 131 \notin P^{+}$