About formulas in extenso for small number of Somas

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In the section with title: The Decomposability of Somas, Carathéodory writes: If none of the decomposition soma S_i is empty, then there are x(p)= 2^{2^p-1} -1 distinct polynomials that can be formed by means of the somas $A_1, \ldots, A_p$. These numbers grow very rapidly: we find that x(2) = 7, x(3) = 127, x(4) = 32767 and x(5) is a ten-digit number. Thus, the formulas for these polynomials cannot be written out in extenso even when the number of somas is small.
Certainly we have a problem of growth but as the enumeration can be done up to isomorphism if you use the automorphism group, in fact we can write in extenso the formulas.

For example for two somas we get 7 formulas but only 5 are different.

S_1, S_2, S_1+S_2, S_2+S_3, $S_1+S_2+S_3$.

S_1 =AB, S_2 = B+AB, S_1+S_2=B, S_2+S_3 = A+B, $S_1+S_2+S_3$.=A+B

For three somas we get 127 formulas but only 39 are different. And except the union A+B+C, the others are complementary so we have only 19 formulas to write and we have done it.