A new probable prime in LR project
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We just found a new probable prime with 672.156 digits. This result is the new step of the Lehmer – Ramanujan Project which started with the study of the odd prime values of the Ramanujan tau function. The tau function is defined as the Fourier coefficients of the modular discriminant
$\Delta(z) = q + \prod_{n=1}^{+\infty}(1 - q^n)^{24} = \sum_{n=1}^{+\infty} \tau(n)q^n$

where z lies in the complex upper half-plane and $q = e^{2\pi iz}$.

We defined $LR(p,n) \coloneqq \tau(p^{n-1})$ and we shown that the odd prime values are of the form $LR(p,q)$ where p, q are odd primes. We proved that if n is a positive integer each that z(n) is an odd prime then n is equal to p^{q-1} where p and q are odd primes and p is ordinary. Murty and al also proved that there exists effectively computable absolute constant c > 0, such that for all positive integers n for which τ(n) is odd, we have $|\tau(n)| \geq (\log n)^c$.
Due to the theory of Lucas sequences we got arithmetical properties and we proved the following theorem. Let p and q be two odd primes, p ordinary. If d is a prime divisor of LR (p,q), then d ≡ ±1 (mod 2q) or d = q. Moreover q|LR(p,q) if and only if q|Dp where Dp = tau$(p)^2$ - $4p^{11}$.

With our estimations we got that the expected number of primes of the form LR (p,q) for prime p fixed and $q < q_{max}$ is

$$\frac{2e^\gamma}{11\log p}\sum_{odd\ prime\ q<q_{max}}\frac{\log 2q}{q} \sim \frac{2e^\gamma \log q_{max}}{11\log p}$$

In this project, we certified many Titan and Giant prime numbers. For example we found 13 new prime numbers with more than 10.000 digits. Our project continued with the research of new probable prime numbers with more than 100.00 digits. And at that time we found 17 hyper giant prime numbers. The newest and biggest one is LR(151, 56087) which has 672.156 digits. In the LR project we have a team with many computers who helps us to continue beyond but any new help is welcomed.